THE UNIVERSITY OF CHICAGO

ESSAYS ON INTERNATIONAL BUSINESS CYCLES

A DISSERTATION SUBMITTED TO

THE FACULTY OF THE DIVISION OF THE SOCIAL SCIENCES

IN CANDIDACY FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

DEPARTMENT OF ECONOMICS

BY

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CHICAGO, ILLINOIS

DECEMBER 1991

ACKNOWLEDGEMENTS

I thank the members of my thesis committee, Professors Michael Woodford (chairman), John Huizinga and José Scheinkman. I am particularly grateful to Professors Michael Woodford and John Huizinga for their encouragement and advice. In addition, I benefited from comments and suggestions by Professors Andrew Atkeson, Marianne Baxter, Yum-keung Kwan and Michael Mussa as well as by my colleagues in Chicago, José Barrionuevo, Eneas Caldino, Chris Erceg, Andy Filardo, Peter Ireland, Youngjae Lim and John Matsusaka. I also would like to thank the members of the Macro Lunch and the International Economic workshop at the University of Chicago and as well as seminar participants at Université du Québec à Montréal, Université de Montréal, Ecole des Hautes Etudes Commerciales (Montréal), University of Southern California, University of California at Riverside, University of Saskatchewan, University of British Columbia, McGill University, Université Laval and the Fed of Philadelphia for helpful comments.

Financial support from the University of Chicago, the Earhart Foundation, the Bradley Foundation, the Sloan Foundation grant to the International Economic Workshop and from Université de Montréal is gratefully acknowledged.

I thank my parents for their support during my studies and I dedicate this thesis to them.

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CHAPTER I

INTRODUCTION

Models of the international economy which assume complete asset markets predict that consumption co-moves closely in different countries as this structure of asset markets allows agents in different countries to 'pool' the country-specific risks which they face (see Scheinkman (1984), Leme (1984)). Examples in this class of models include the recent international Real Business Cycle models of, among others, Backus, Kehoe & Kydland (1989), Baxter & Crucini (1989), Stockman & Tesar (1991).

The first essay in this thesis (chapter II) tests the implications for the trend behavior of consumption of models of the international economy which assume complete asset markets. In a world where consumptions and real bilateral real exchange rates of different countries follow unit root processes, such models predict that (under certain assumptions about preferences) consumptions and bilateral real exchange rates are cointegrated for any given pair of countries. The paper presents statistical tests which suggest that data on consumptions and real exchange rates for the US, Japan, France, Britain, Italy and Canada during the period 1971-1987 are inconsistent with this prediction.

These findings suggest that models with limitations on international asset markets might be needed to explain the international covariation of consumption.

The second essay in this thesis (chapter III) presents a Real Business Cycle model in which limitations on international capital markets exist in the sense that only debt contracts are available for international capital flows. Simulations of the model suggest that it can explain the low cross-country correlations observed in detrended consumption data, and that for 'realistic' cross-country correlations of the exogenous shocks.

The second essay argues also that a model which allows for *additive* technology shocks is better able to explain the observed positive correlations of investment and output across countries than standard business cycle theories in which *multiplicative* shocks to total factor productivity are the only source of economic fluctuations. One possible interpretation of the *additive* shocks is as shocks to government consumption.

CHAPTER II

THE STRUCTURE OF INTERNATIONAL CAPITAL MARKETS AND COMMON TRENDS IN INTERNATIONAL CONSUMPTION DATA: AN EMPIRICAL ANALYSIS

1. Introduction

Models of the international economy which assume complete asset markets predict that consumption co-moves closely in different countries as this structure of asset markets allows agents in different countries to 'pool' the country-specific risks which they face (see Scheinkman (1984), Leme (1984)). Examples in this class of models include the recent international Real Business Cycle models of, among others, Backus, Kehoe & Kydland (1989), Baxter & Crucini (1989), Stockman & Tesar (1991).[1]
Previous empirical evaluations of these models have typically focused on their high-frequency implications (see , e.g., Backus, Kehoe & Kydland (1989) who 'calibrate' their model of international business cycles by comparing its predictions to detrended data). In contrast the present paper tests long-run implications of these models, by focusing on their implications for the trend behavior of consumption in different countries.

Unit root tests suggest that consumption in the US, Japan, France, Britain, Italy and Canada during the period 1971-1987 can be described by unit root processes. Under certain assumptions about preferences, models with complete asset markets predict that in a setting where (log) consumptions and bilateral real exchange rates follow unit root processes, these

[1]See these papers and chapter III for further references.

variables are (stochastically) cointegrated for any given pair of countries, i.e., there exists a linear combination of these series which is trend stationary (see Ogaki (1988)). The paper presents results based on two types of cointegration tests (Park (1991), Phillips & Ouliaris (1990)) which suggest that consumption behavior in the countries mentioned earlier is inconsistent with this prediction.

These results suggest that models with incomplete asset markets might be needed to explain the international covariation of consumption. The paper considers a structure of incomplete asset market in which consumptions in different countries fail to be cointegrated. What makes asset markets incomplete in this economy is the fact that only debt instruments can be used for international borrowing and lending. When rates of change of consumption and of real exchange rates are log-normally distributed and when agents have iso-elastic preferences, this model implies a linear restriction on conditional expectations of future rates of consumption and real exchange rates in different countries. Various tests of this restriction are discussed. Simulations of an international Real Business Cycle model with the asset market incompleteness which was just described suggest that it can explain the low cross-country correlations observed in detrended consumption data (see chapter III). Generalized Method of Moments based tests of a version of the model with incomplete asset markets which assumes a single consumption good fail to reject the model. Tests which are based on a method due to Velu et al. (1986) reject the single good version of the model, but they are more supportive of a version of the model in which countries consume heterogeneous (country-specific) consumption goods.

Section 2 of the paper presents a model of the international economy with complete asset markets and it describes its testable implications. Section 3 describes statistical methods used to test the predictions of that model. Section 4 describes the data used for the tests and section 5 presents the empirical results. Section 6 summarizes the findings obtained in sections 2-5. Section 7 presents a model with an alternative (incomplete) asset markets structure and it tests implications of this asset structure. Tables with summary statistics and empirical results are presented in the appendix.

2. Complete International Asset Markets and Comovements in International Consumption

In the presence of complete asset markets, the behavior of the world economy can be characterized as the solution to a social planning problem which consists in maximizing a weighted sum of the utility levels of the (representative) consumers of the different countries subject to world resource and technology constraints (see , e.g., Scheinkman (1984), Leme (1984), Backus, Kehoe & Kydland (1989), Baxter & Crucini (1989), Stockman & Tesar (1990), Yi (1990)).[2]

Consider a world with 'I' countries indexed by i=1,...,I. Following Backus, Kehoe & Kydland (1989), assume that each country is inhabited by a (representative) agent who is infinitely lived and whose intertemporal preferences can be represented by a time separable utility function of the

[2]Applications of the same idea in a context which involves individual consumers in a closed economy setting can be found in Cochrane (1991), Mace (1991) and Townsend (1989).

form $V_t^i=E_t\sum_{\tau=0}^{\tau=\infty}(\beta^i)^\tau \upsilon_{t+\tau}^i$; here E_t denotes expectations conditional on informations available in period t; $0<\beta^i\equiv 1/(1+b^i)<1$, where b^i is i's subjective rate of time preference and '$\upsilon_{t+\tau}^i$' is the agent's instantaneous utility function in period $t+\tau$.

The social planning problem mentioned earlier consists in maximizing the term $\sum_{i=1}^{i=I}\lambda^i V_0^i$, where λ^i is a 'welfare weight' attached to country i. The welfare weights are time invariant. They reflect the distribution of wealth between the different countries.[3]

2.1. *A World with a Single Consumption Good*

Assume that--as in the Real Business Cycle models of, among others, Backus, Kehoe & Kydland (1989) and Baxter & Crucini (1989)--there exists a unique consumption good which can costlessly be shipped between countries, and let C_t and c_t^i denote total world-wide consumption and the consumption of country 'i' in period t respectively.

Let C_t^* denote the optimal aggregate world consumption implied by the solution of the social planning problem. Given optimal world consumption, the optimal consumptions of countries $i=1,\ldots,n$ can be characterized by solving the following problem:

Max $\sum_{i=1}^{i=n}\lambda^i V_0^i$ by choice of $c_t^1,\ldots,c_t^n$ s.t. $\sum_{i=1}^{i=n}c_t^i\le C_t^*$ for all periods $t\ge 0$.

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[3]To any competitive equilibrium in the world economy with complete asset markets there corresponds a set of welfare weights which is such that the solution of the social planning problem is identical to the competitive equilibrium. Countries whose wealth (evaluated at the prices which obtain in a given competitive equilibrium) is large are given large welfare weights in the social planning problem.

Throughout this paper, it is assumed that the period t utility function of country i is additively separable in consumption and all other goods (denoted by 'Z') which affect i's well-being and that it depends on consumptions in period t only (departures from these assumptions are discussed below):

$$\upsilon^i_t=u^i(c^i_t)+\varphi^i(Z^i_t) \ , \tag{2.1}$$

where u^i is assumed to be an increasing and concave function.

For this specification of preferences, the solution to the social planning problem requires the following condition to be satisfied:

$$\lambda^i*(\beta^i)^t*u^{i\prime}(c^i_t)=\lambda^j*(\beta^j)^t*u^{j\prime}(c^j_t) \ . \tag{2.2}$$

(2.2) imposes strong restrictions on the behavior of consumption in countries i and j: it implies that country j's consumption can be expressed as an increasing function of country i's consumption.[4]

To obtain testable implications from (2.2), I follow the Real Business Cycle literature and adopt a constant elasticity specification of the period utility functions:

$$u^k=A^k*(1/\sigma^k)*c^{\sigma^k}, \text{ with } A^k>0, \ \sigma^k<1 \quad \text{for } k=i,j \tag{2.3}$$

With these preferences, condition (2.2) becomes

$$\lambda^i*(\beta^i)^t*A^i*(c^i_t)^{(\sigma^i-1)}=\lambda^j*(\beta^j)^t*A^j*(c^j_t)^{(\sigma^j-1)}. \tag{2.4}$$

[4]This follows from the concavity of υ^i and υ^j. Recall that the weights λ^i and λ^j are time invariant.

Taking logs of (2.4) yields the expression

$$(\sigma^{i}-1)*\ln(c_t^{i})=K^{i,j}+\ln(\beta^{j}/\beta^{i})*t+(\sigma^{j}-1)*\ln(c_t^{j}), \tag{2.5}$$

where $K^{i,j}\equiv\ln[(\lambda^{j}*A^{j})/(\lambda^{i}*A^{i})]$.

(2.5) forms the basis for the first set of tests to be presented below.

Statistical tests discussed below suggest that consumption series in the sample of countries considered in this paper can be modeled as unit root processes.

(2.5) implies that there exists a linear combination of $\ln(c^{i})$ and $\ln(c^{j})$ which *exactly* equals a deterministic trend. From an empirical point of view however it makes more sense, to interpret (2.5) as a long-run 'equilibrium condition' rather than as a condition which each period is exactly satisfied in the data.[5] Given the evidence according to which (log) consumptions follows unit roots processes, I interpret condition (2.5) as predicting that $\ln(c^{i})$ and $\ln(c^{j})$ are 'stochastically cointegrated', i.e., that there exists a linear combination of these variables which is trend stationary.[6] Ogaki (1988) defines random

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[5]For example because of measurement errors in consumption data. Random taste shocks are another reason why the deterministic restriction on the consumptions of different countries specified in (2.5) may fail to hold. To model these shocks, assume for example that A^{i} and A^{j} are random numbers rather than constant parameters, as was assumed up to now. Then $K^{i,j}$ is random. Under the identifying assumption that $K^{i,j}$ is stationary, condition (2.5) predicts that a linear combination of the consumptions of countries i and j is trend stationary.

[6]A variable is trend stationary if it can be represented as the sum of a deterministic trend and a covariance stationary random variable.

variables with unit roots for which there exists a linear combination which is trend stationary as "stochastically cointegrated" . He contrasts this with the concept of "deterministic cointegration" which requires the existence of a linear combination which is covariance stationary.[7]

The tests presented below are tests of the joint hypothesis of complete markets and the preference specifications (2.3). It should however be noted that - up to a (log) linear approximation - linear restrictions similar to (2.5) follow from the risk-sharing condition (2.2) for more general specifications of the instantanous utility function 'u'. Under certain conditions, restrictions on the behavior of consumption in different countries which are simliar to (2.5) can also be obtained if the assumption that life-time utility functions are time-separable is dropped.[8]

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[7]See Campbell & Perron (1991) for further discussions of these concepts.

[8]Assume for example that $u_t^k=u^k(c_t^k+a*c_{t-1}^k)$ (where a is a constant), i.e., that the period t instantaneous utility of country k depends on consumptions in periods t and t-1. The solution to the social planning problem now requires the following condition to be satisfied for all periods t≥0:

$$\lambda^i*\left((\beta^i)^t*u^{i\prime}(c_t^i+a*c_{t-1}^i)+(\beta^i)^{t+1}*a*E_t u^{i\prime}(c_{t+1}^i+a*c_t^i)\right)=$$
$$\lambda^j*\left((\beta^j)^t*u^{j\prime}(c_t^j+a*c_{t-1}^j)+(\beta^j)^{t+1}*a*E_t u^{j\prime}(c_{t+1}^j+a*c_t^j)\right).$$

This condition is satisfied if

$$\lambda^i*(\beta^i)^\tau*u^{i\prime}(c_\tau^i+a*c_{\tau-1}^i)=\lambda^j*(\beta^j)^\tau*u^{j\prime}(c_\tau^j+a*c_{\tau-1}^j) \qquad \text{(N.1)}$$

holds in all periods.

(to obtain this condition it is important that the parameter 'a' is the same for countries i and j). For the constant elasticity specification (2.3), this condition implies a linear restriction on the behavior of $\ln(c_t^i+a*c_{t-1}^i)$ and $\ln(c_t^j+a*c_{t-1}^j)$. A more convenient restriction can be

The constant elasticity specification (2.3) is a standard feature of many model in macroeconomics and finance. Existing Real Business Cycle models use it, as it is the only preference specification which - in a model with infinitely lived agents - yields steady state growth paths for which consumption growth rates and real interest rates are constant.

In this context it seems important to note that the Real Business Cycle literature typically allows for preferences in which consumption and work effort interact in a non-separable way; when labor is immobile internationally, non-separabilities between consumption and work effort reduce the international correlation of consumption.[9] This literature assumes however that there exists a steady state level of work effort which

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obtained if instead of a constant elasticity utility function an exponential utility function is used: $u^k(c_t^k+a*c_{t-1}^k)=A^k*\exp(B^k*(c_t^k+a*c_{t-1}^k))$ for k=i,j, where A^k and B^k are constants. Rewriting (N.1) for this utility function and taking logs of the resulting expression yields:

$$B^i*(c_t^i+a*c_{t-1}^i)=\kappa^{i,j}+\ln(\beta^j/\beta^i)*t+B^j*(c_t^j+a*c_{t-1}^j), \qquad (N.2)$$

where $\kappa^{i,j}\equiv\ln[(\lambda^j*A^j*B^j)/(\lambda^i*A^j*B^j)]$. Assume that $\{c_t^i\}$ and $\{c_t^j\}$ follow unit root processes, i.e. that $c_t^k=\mu^k+c_{t-1}^k+\varepsilon_t^k$ for k=i,j where ε_t^k is a mean zero variable which is covariance stationary. Substituting c_{t-1}^i and c_{t-1}^j from these expressions into (N.2) yields:

$$B^i*c_{t-1}^i=\hat{\kappa}^{i,j}+B^j*c_{t-1}^j+\ln(\beta^i/\beta^j)*t+\eta_t, \qquad (N.3)$$

where $\hat{\kappa}^{i,j}\equiv\kappa^{i,j}+B^i*\mu^i-B^j*\mu^j$ and $\eta_t\equiv B^j*\varepsilon_t^j-B^i*\varepsilon_t^i$ is a covariance stationary random variable. According to (N.3) consumptions in countries i and j are cointegrated.

[9] See Devereux, Gregory & Smith (1991) for detailed discussions of this point.

is constant.[10] Under this assumption, the restriction for the trend behavior of consumption in different countries which are implied by (2.5) continue to hold.[11]

In addition to the setting with a single consumption good, I also consider economies in which countries differ in the consumption goods which they consume. This is motivated by the large variations in real exchange rates experienced by the countries in the sample. Note for example that on average the price of a unit of non-durable Japanese consumption in terms of US consumption has risen at the rate of 5.4% p.a. during the period 1971:1-88:1.[12] Table 2 formally tests the hypothesis that unconditional means of the log growth rates of real exchange rates are zero. For most country pairs in the sample there is strong evidence against this hypothesis.

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[10] See , e.g., King, Plosser & Rebelo (1988).

[11] This can easily be illustrated using the standard utility function used in the Real Business Cycle literature (see, e.g., King, Plossser & Rebelo (1988), Rotemberg & Woodford (1989)): $u=A*(c*\exp[-\psi(L)])^{\sigma}$ where 'L' denotes the number of hours worked and ψ is an increasing function. For this utility function, condition (2.5) becomes:
$(\sigma^{i}-1)*\ln(c_t^{i})-\sigma^{i}*\psi^{i}(L_t^{i}) =K^{i,j}+\ln(\beta^{j}/\beta^{i})*t+(\sigma^{j}-1)*\ln(c_t^{j})-\sigma^{j}*\psi^{j}(L_t^{j})$.
If $\{L_t^{i}\}$ and $\{L_t^{j}\}$ are covariance stationary, then the prediction that $\ln(c_t^{i})$ and $\ln(c_t^{j})$ are stochastically cointegrated continues to hold (actually, the prediction continues to hold provided $\psi^{i}(L_t^{i})$ and $\psi^{j}(L_t^{j})$ that are trend stationary).

[12] Real exchange rates defined in terms of aggregate consumption have behaved similarly during the sample period.

2.2. *A World with one Country-Specific Good*

I next consider a model in which each country consumes a single good which is distinct from the goods consumed by other countries. In what follows, I refer to this framework as the 'one country-specific good model'.[13] In this setting, the assumption of complete asset markets implies that

$$\lambda^{i}*(\beta^{i})^{t}*u^{i,}=\lambda^{j}*(\beta^{j})^{t}*u^{j,}*R_{t}^{i,j} \tag{2.6}$$

holds where $R_t^{i,j}$ is the relative price of the consumption goods of countries i and j (i.e. $R^{i,j}$ is the bilateral real exchange rate of these countries in terms of their respective consumption goods: let p_t^i and p_t^j denote the prices of c_t^i and c_t^j respectively in terms of some numéraire; then $R_t^{i,j}\equiv p^i/p^j$).

To understand why (2.6) is implied by the assumption of complete asset markets, note that the marginal rate of substitution between c_t^i and c_t^j from the point of view of the social planner[14] is $[\lambda^{i}*(\beta^{i})^{t}*u^{i,}]$ / $[\lambda^{j}*(\beta^{j})^{t}*u^{j,}]$. (2.6) is the condition that the planner's marginal rate of substitution between c_t^i and c_t^j is equated to the relative price of these two goods.[15]

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[13] A more realistic specification would assume that each country consumes *two* goods: a tradable good (which is consumed by more than one country) and a non-tradable good ('home' good). The framework with one country-specific consumption good is adopted because available statistical sources do not disaggregate national consumption into home and imported goods.

[14] Recall that the planner's objective function is $\sum_{\tau=0}^{\tau=\infty}\lambda^{i}*V_0^i$.

[15] A more intuitive justification of condition (2.6) is to imagine that there exists a material input (such as oil) which is freely traded between countries and which is used by all countries as an input into the

With the constant elasticity preferences defined in (2.3), condition (2.6) implies that

$$(\sigma^i-1)*\ln(c_t^i)=K^{i,j}+\ln(\beta^j/\beta^i)*t+(\sigma^j-1)*\ln(c_t^j)+\ln(R^{i,j}) \ , \tag{2.7}$$

where as before $K^{i,j}\equiv\ln[(\lambda^i*A^j)/(\lambda^i*A^i)]$.

As shown below, bilateral real exchange rates in the sample of countries can be described by unit root processes. I therefore interpret (2.7) as predicting that (log) consumptions and (log) bilateral real exchange rates are stochastically cointegrated for these countries.

2.3. *A World with Two Country-Specific Goods*

I also consider an extension of the previous model in which each country consumes two country-specific goods: assume that country i's period t utility function is

$$u_t^i=u^i(nd_t^i,s_t^i), \tag{2.8}$$

where nd_t^i and s_t^i are the two goods consumed by i. In the empirical part of the paper these two goods will be interpreted as non-durables and services

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production of their country-specific consumption good. Denote this good by 'z'. At the optimum the social planner is indifferent between allocating an additional unit of the tradable input to country i and allocating that unit to country j. Allocating an additional unit to country k=i,j allows to increase that country's production (and consumption) of its country-specific consumption good by $\partial c^k/\partial z$ (the marginal product of the material input). Hence the solution to the social planning problem has the property that

$$\lambda^i*(\beta^i)^t*(\partial c^i/\partial z)*(\partial u^i(c^i)/\partial c^i)=\lambda^j*(\beta^j)^t*(\partial c^j/\partial z)*(\partial u^j(c^j)/\partial c^j).$$

Under perfect competition, $\partial c^k/\partial z=1/p^k$ holds where p^k is the price of country k's consumption good in terms of the material input; we therefore have that $\lambda^i*(\beta^i)^t*(\partial u^i(c^i)/\partial c^i)=\lambda^j*(\beta^j)^t*(p^i/p^k)*(\partial u^j(c^j)/\partial c^j)$, i.e. (2.6) holds.

consumptions respectively. (2.8) is motivated by Stockman & Tesar (1990) who present a Real Business Cycle model which disaggregates private consumption into tradables and non-tradables (services);[16] they argue that this goods markets structure helps the model explain the low cross-country correlations which are observed in detrended consumption data.

In the world with two country specific consumption goods, the solution of the social planning problem satisfies the following conditions:

$$\lambda^{i}*(\beta^{i})^{t}*u^{i}_{nd}(nd^{i}_{t},s^{i}_{t})=\lambda^{j}*(\beta^{j})^{t}*u^{j}_{nd}(nd^{j}_{t},s^{j}_{t})*RND^{i,j}_{t}\ , \qquad (2.9\ a)$$

and $$\lambda^{i}*(\beta^{i})^{t}*u^{i}_{s}(nd^{i}_{t},s^{i}_{t})=\lambda^{j}*(\beta^{j})^{t}*u^{j}_{s}(nd^{j}_{t},s^{j}_{t})*RS^{i,j}_{t}\ , \qquad (2.9\ b)$$

where $RND^{i,j}$ ($RS^{i,j}_{t}$) is the relative price of the nd (s) goods consumed by countries i and j.

The tests of (2.9 a) and (2.9 b) use a constant elasticity utility function:

$$u^{k}(nd^{k},s^{k})=A^{k}*(nd^{k})^{\sigma^{k}}*(s^{k})^{\mu^{k}}\ . \qquad (2.3')$$

This utility function is increasing and concave in nd and s iff $\sigma+\mu<1$ and $\sigma*\mu>0$ hold.

Writing (2.9 a) and (2.9 b) using the constant elasticity function specified in (2.3') and taking logs yields the following cointegrating relations:

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[16]Empirical research in international economics has frequently assumed that services are non-tradable: see for example Kravis, Heston & Summers (1982), Kravis & Lipsey (1983, 1988).

$$(\sigma^i-1)*\ln(nd_t^i)+\mu^i*\ln(s_t^i)= \qquad (2.10\ a)$$

$$K^{i,j}+\ln(\beta^j/\beta^i)*t+(\sigma^j-1)*\ln(nd_t^j)+\mu^j*\ln(s_t^j)+\ln(RND_t^{i,j});$$

$$\sigma^i*\ln(nd_t^i)+(\mu^i-1)*\ln(s_t^i)= \qquad (2.10\ b)$$

$$K^{i,j}+\ln(\beta^j/\beta^i)*t+\sigma^j*\ln(nd_t^j)+(\mu^j-1)*\ln(s_t^j)+\ln(RS_t^{i,j}),$$

where as before $K^{i,j}\equiv\ln[(\lambda^i*A^j)/(\lambda^i*A^i)]$.

Hence the model with two country-specific consumption goods implies that log consumptions of non-durables and services in countries i and j and the bilateral log real exchange rates between these countries are cointegrated.

Empirically, tests of (2.10 a) and (2.10 b) yield very similar results. The discussion in the rest of the paper focuses on condition (2.10 a).

To conclude this section, I restate the cointegrating relations which will be tested below:

(i) The model with a single consumption good:

$$(\sigma^i-1)*\ln(c_t^i)=K^{i,j}+\ln(\beta^j/\beta^i)*t+(\sigma^j-1)*\ln(c_t^j). \qquad (2.5)$$

(ii) The model with one country-specific consumption good:

$$(\sigma^i-1)*\ln(c_t^i)=K^{i,j}+\ln(\beta^j/\beta^i)*t+(\sigma^j-1)*\ln(c_t^j)+\ln(R^{i,j}) . \qquad (2.7)$$

(iii) The model with two country-specific consumption goods:

$$(\sigma^i-1)*\ln(nd_t^i)+\mu^i*\ln(s_t^i)= \qquad (2.10\ a)$$

$$K^{i,j}+\ln(\beta^j/\beta^i)*t+(\sigma^j-1)*\ln(nd_t^j)+\mu^j*\ln(s_t^j)+\ln(RND_t^{i,j}),$$

The next two sections discusses the statistical methods and the data which will be used for the tests of these cointegrating relations.

3. Statistical methods

I assume that all variables can be represented as sums of deterministic trends and mean zero stochastic components as in the following expression:[17]

$$x_t=a+b*t+Z_t \;, \tag{3.1}$$

where Z_t is a mean zero random variable. Following Campbell & Perron (1991), I assume that Z_t follows an ARMA process: $A(L)Z_t=B(L)*\varepsilon_t$, where A(L) and B(L) are polynomials in the lag operator L, while ε_t is iid. The series $\{x_t\}$ has a unit root if the autoregressive polynomial of Z_t has one unit root, while all other roots are strictly outside the unit circle (see Campbell & Perron (1991)). If this condition is satisfied, $(1-L)*x_t$ is covariance stationary with mean b.

3.1. *Unit Root Tests*

Several unit root tests have been discussed in the econometrics literature.[18] The test used here is the Augmented Dickey-Fuller (ADF) test which consists in estimating the following model by OLS:

$$\Delta x_t=\alpha+\beta*t+\phi*x_{t-1}+\sum_{s=1}^{k}\varphi_k*\Delta x_{t-k}+\varepsilon_t. \tag{3.2}$$

$\phi=0$ holds under the null hypothesis that the stochastic component of 'Z' in

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[17] Campbell & Perron (1991) discuss time series with more general types of deterministic components, including ones characterized by variations in intercepts, slopes etc.

[18] See Campbell & Perron (1991) and Cochrane (1991) for references.

(3.1) follows an ARIMA(k+1,1,0) process.[19] The linear trend $\alpha+\beta^*t$ is included in (3.2), because the deterministic part of x_t is assumed to be a linear time trend (see (3.1)).

Under the hypothesis that $\phi=0$, the distribution of the studentized value of the OLS estimate of ϕ is non-standard; critical values for this distribution are tabulated in Fuller (1976).

Results from unit root tests are discussed in section 5.1.

3.2. *Cointegration Tests*

I use two methods for testing for cointegration: the 'spurious regression' test proposed by Park (1990) and the 'residual based' test of Phillips & Ouliaris (1990).

Park's method allows to test the null hypothesis that a set of variables is cointegrated. This is an attractive feature as it allows to directly test the cointegrating relations implied by the models discussed in the last section. In contrast, the Phillips & Ouliaris test (like all other cointegration tests currently available in the econometrics literature)[20] tests the hypothesis that a set of variables is *not* cointegrated. Phillips & Ouliaris tests are reported because failure to reject the no-cointegration hypothesis would provide useful information on the model

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[19] As ARIMA(p,1,q) processes can be approximated by ARIMA(g,1,0) processes for suitable choices of 'g', the Dickey-Fuller testing procedure can also be applied when Z_t has a moving average component (see Said & Dickey (1984)), although using the Dickey-Fuller procedure can be problematic if the MA parameters are large (see Schwert (1987)).

[20] See Campbell & Perron (1991) for an overview.

which is tested.

3.2.1. **Park's (1990) test**[21]

Assume that the q+1 variables $x_t^0, x_t^1, \ldots, x_t^q$ all have unit roots. If they are cointegrated, the residual in the following cointegrating regression is stationary:

$$x_t^0 = D + F*t + \sum_{s=1}^{s=q} \psi_s * x_t^s + \eta_t \quad . \tag{3.3}$$

Park's method considers a variant of this cointegrating regression which obtains when the 'x' variables in (3.3) are transformed by adding to them certain stationary random variables (see Park (1990), p.117). Denote the transformed 'x' variables by $\hat{x}$. Park (1991) refers to the cointegrating regression which obtains when in (3.3), the x variables are replaced by $\hat{x}$ as a 'canonical cointegrating regression'. To apply Park's method, 'superfluous' regressors, such as high order time polynomials,[22] are added to the canonical cointegrating regression and one tests whether these superfluous regressors enter significantly in the regression.

For example, let $z_t = (t^2, t^3, \ldots, t^p)$ and let γ be a column vector of coefficients which is conformable with z_t. To apply Park's method, one can add the term $z_t * \gamma$ to the canonical cointegrating regression:

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[21] See Campbell & Perron (1991) and and Fisher & Park (1990) for useful discussions of this test.

[22] Park (1990) shows that his test can also be performed by adding superfluous random variables with unit roots (such as computer generated random walks) to the cointegrating regression.

$$\hat{x}_t^0 = D + F*t + \sum_{s=1}^{s=n} \psi_s * \hat{x}_t^s + z_t * \gamma + \hat{\eta}_t \tag{3.4}$$

Park's test exploits the fact that if x_t^0, $x_t^1, \dots,$ x_t^q are stochastically cointegrated, then the Wald test statistic for the hypothesis $\gamma=0$ converges to a stable limiting distribution. Otherwise, the Wald statistic diverges. The Park test statistic for the test of the null of cointegration is a transformation of the Wald test statistic.

As the outcome of the Park test can depend on which of the 'x' variables is used on the left-hand side of the cointegrating regression (3.3), test results will be reported for all possible choices for the left-hand side variable in that equation.

Park (1991) shows how to conduct tests of hypotheses concerning the coefficients of the cointegrating regression. This is useful, as the coefficients of the cointegrating relations (2.5), (2.7) and (2.10 a), are functions of preference parameters. Park's method therefore allows to test whether the preference parameters implied by the estimated coefficients of the cointegrating regressions are consistent with well-behaved utility functions.

To compute Park's test statistics for the test of the null of cointegration, it is necessary to correct for serial correlation in the residual in the cointegrating relation (3.3) and in the first differences of the 'x' variables included on the right-hand side of (3.3) (see Park (1990), p.117) . I use the Newey & West (1987) method and 10 autocorrelations for that purpose.[23]

The Park tests reported below use t^2, t^3 and t^4 as 'superfluous' regressors in (3.4).

3.2.2. The Phillips & Ouliaris (1990) method

The fact which underlies the 'residuals based' method is that if the n+1 variables $x_t^0, x_t^1, \ldots, x_t^q$ all have unit roots, and if they are *not* stochastically cointegrated, then the residual in the cointegraing regression (3.3) is non-stationary; hence unit root tests can be applied to that regression residual in order to test the null hypothesis that $x_t^0, x_t^1, \ldots, x_t^q$ are *not* cointegrated. Phillips & Ouliaris (1990) use Phillips' (1987) $\hat{Z}_\alpha$ and $\hat{Z}_t$ unit root test statistics for this purpose. In the work presented below, I use both of these tests.

To compute the $\hat{Z}_t$ and $\hat{Z}_\alpha$ test statistics, one runs the regression: $\hat{\eta}_t = \alpha * \hat{\eta}_{t-1} + \hat{k}_t$, where $\hat{\eta}_t$ is the regression residual obtained by fitting (3.3). Under the null hypothesis that the variables x_t^0, $x_t^1, \ldots, x_t^n$ are not cointegrated, $\alpha=1$ holds. The $\hat{Z}_\alpha$ test statistic is a transformation of the expression $T*(\hat{\alpha}-1)$ (where $\hat{\alpha}$ is the OLS estimate of α and T is the sample length) while the $\hat{Z}_t$ test is a transformation of the t-test statistic for a test of the hypothesis that $\alpha=1$.

To compute the Phillips & Ouliaris test statistics, it is necessary to correct for serial correlation in the residual $\hat{k}_t$ (see p.171 in Phillips

[23]Table 7 reports the results of Monte Carlo simulations which suggest that for the available sample length (69 periods) it might be appropriate to correct for 10 autocorrelations.

& Ouliaris (1990)). I use the Newey & West method and 10 autocorrelations for this purpose.
As outcomes of the Phillips & Ouliaris test can depend on which 'x' variable is used on the left-hand side in the cointegrating regression (3.3), test results will be reported for all possible choices for the left-hand side variable in that equation.

4. The Data

This paper uses quarterly data on private consumption in the US, Japan, France, Britain, Italy and Canada (henceforth I refer to these countries as 'G6' countries)[24] from the OECD Quarterly National Accounts (QNA) database. The consumption series for the US, France, Italy and Canada are supplied in seasonally adjusted form by the QNA. Series for Japan and the UK are provided in seasonally unadjusted form; I used the 'esmooth' command in the econometrics package RATS to seasonally adjust them. All consumption figures used in the empirical work are expressed in per capita terms.[25] The consumption data are available for the period 1970:1-88:1 (71:1-88:1 for Italy). The database contains non-durable and services consumption series in current prices (in units of the respective national currencies) as well as in constant prices; by dividing the former by the latter, I constructed price indices for non-durables and for services. Exchange rate data are

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[24]The G6 equals the more familiar G7 without Germany. Germany is not included in the sample because the OECD quarterly national accounts database does not provide data on German consumption of non-durables and services.

[25]Using population figures from the International Financial Statistics database.

taken from the International Financial Statistics database.
Descriptive statistics on consumption and real exchange rates for the G6 countries are presented in tables 1-4.

I tested the cointegrating relations implied by the single consumption good model (see 2.5) and by the model with one country-specific consumption good (see 2.7) for a variety of consumption measures: (i) non-durables, (ii) non-durables plus services, (iii) non-durables plus services and government consumption, (iv) total private consumption expenditures (including expenditures on durables), (v) total private consumption expenditures plus government consumption expenditures.
It appears that the tests results using these different consumption measures are quite similar.
For the tests of the single good model and of the model with one country-specific good, results are reported below for a consumption measures consisting of non-durables plus services, i.e. the cointegrating relations (2.5) and (2.7) are tested using the sum of non-durables and services consumption in country i as a measure for the consumption good 'c'. The tests of the model with two country-specific consumption goods identify these two goods with non-durables and services respectively.

5. Empirical Results

5.1. *Results From Unit Root Tests*

Tables 5 and 6 present the results of unit root tests for consumptions and bilateral real exchange rates. These tests were conducted for the following values of the lag parameter 'k' in (3.2): k=0,1,2,3,4,5,6.

Table 5 presents unit root tests for consumption. For countries other than Japan we see that there is little evidence at the 10% level against the unit root hypothesis. For the Japanese series, the table provides strong evidence against the unit root hypothesis; it appears however that for larger values of k there is little evidence against the unit root hypothesis. Because of this, I do not exclude Japan from the sample.

Table 6 presents tests of the hypothesis that the bilateral real exchange rates (in logs) of the G6 countries follow unit root processes. The tests are for real exchange rates in terms of non-durable consumption goods (test results for real exchange rates in terms of services are similar and are therefore not been presented in the appendix). Even at the 50% level, there is little evidence against the hypothesis that bilateral real exchange rates follow unit root processes.

5.2 *Results From Cointegration Tests*

5.2.1. **Park tests**

Table 8 reports results for the Park test.

For each country pair, table 8 reports p-values of 2 Park test statistics for the single good model (corresponding to two different choices for the left-hand side variable in the cointegrating regression (3.3)) and hence there is a total of 30 test statistics for the single good model.[26] 19 of the 30 test statistics reject the null of cointegration at the 20% level and 12 test statistics reject the null at the 10% level.

Concavity of the utility functions of countries i and j implies that $(\sigma^i-1)/(\sigma^j-1)>0$. With few exceptions, this inequality is satisfied by the data.[27]

The test results threfore cast doubt on the single good model.[28] They support the findings of Neusser (1991), who argues that a consumption measure consisting of the sum of private and government consumption fails to be deterministically cointegrated for Austria, Canada, Germany, Japan, Britain and the US.[29]

Table 8 also reports test results for the model with one country-specific consumption good. A total of 45 test statistics is now reported (for each country pair, three test statistics are computed). The cointegrating

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[26] There are 15 country pairs in the sample.

[27] A '(P)' in the table indicates a statistically significant (at the 5% level) violation of the condition $(\sigma^i-1)/(\sigma^j-1)>0$.

[28] A potential qualification of this suggested conclusion from Park's test is the fact that the for many country pairs the choice of the left-hand side variable in the the cointegrating regression affects the outcome of the test: there are 9 country pairs where one of the two test statistics rejects the null of cointegration (at the 20% level), while the the other test statistic fails to reject it. The dependence of the test outcome on the choice of the left-hand side variable is somewhat worrisome because asymptotically that choice does not affect the distribution of Park's test statistic (the asymptotic distribution is used to calculate the p-values reported in table 8).

[29] Neusser uses a cointegration test due to Johansen (1989).

relations implied by the model with one country-specific consumption good are rejected (at the 20% level) by 30 of the 45 test statistics (at the 10% level, there are 20 rejections).

The preference parameters recovered from the cointegrating regressions frequently violate concavity. If we eliminate the cases where for at least one of the countries included in a given country pair, we reject the hypothesis that $\sigma \leq 1$ at the 5% level, we are left with 12 cases where at the 10% level the Park test fails to reject the null of cointegration.

Finally, Table 8 reports test results for the model with two country-specific consumption goods. For each country pair, five test statistics are now computed (corresponding to five different choices of the left-hand side variable in (3.3)) which gives a total of 75 test statistics. At the 20% level, the null-hypothesis of cointegration is rejected for 40 of the 75 test statistics. At the 10% level there are 20 rejections. Statistically significant violations of the conditions $\sigma^{i}+\mu^{i}<1$, $\sigma^{i}*\mu^{i}>0$, $\sigma^{j}+\mu^{j}<1$, $\sigma^{j}*\mu^{j}>0$ [30] occur in approximately two-thirds of the 75 cases considered in table 8 for the model with two country-specific consumption goods.

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[30] $\sigma+\mu<1$ and $\sigma*\mu>0$ are necessary and sufficient conditions under which the utility function in the model with two country-specific goods is concave and increasing in both goods.

5.2.2. **Phillips & Ouliaris tests**

Phillips & Ouliaris test results are in table 10.

The Phillips & Ouliaris method tests the null hypothesis that a set of variables is *not* cointegrated. The test results reported in table 10 are consistent with the hypothesis that the three cointegrating relations derived in section 2 do *not* hold: at the 20% significance level, approximately 80% of the test statistics reported in table 10 for the cointegrating relations (2.5), (2.7) and (2.10 a) fail to reject the hypothesis that the cointegrating relations stated at the end of section 1 do *not* hold.[31]

6. Summary of Empirical Results

Cointegration tests were used to test a model of the international economy which assumes complete asset markets. A version of that model which assumes a single consumption good and iso-elastic utility functions predicts that log consumptions in different countries are cointegrated.

In a world in which each country consumes a country specific consumption good, the log consumptions of different countries and their log bilateral real exchange rates are predicted to be stochastically cointegrated. Finally, a model was considered in which each country consumes two

[31] The proportions of test statistics which yield rejections at the 20% level differ somewhat for the three cointegrating relations. The proportions of rejections by the $\hat{Z}_{\alpha}$ statistic of the null hypothesis of no cointegration are (at the 20% level): 0.20, 0.11 and 0.18 for the single good model, the model with one country-specific good and the model with two country-specific goods respectively. The corresponding proportions of rejections yielded by the Z_t statistic are 0.23, 0.13 and 0.28 respectively.

country-specific goods. In such a setting, a model with complete asset markets implies that (with isoelastic preferences) the consumption of the two country-specific goods by different countries and their bilateral real exchange rates are stochastically cointegrated.

Tests using methods developed by Park (1990) and Phillips & Ouliaris (1990) cast strong doubt on these predictions.

The results of this paper suggest that the failure of the single good model cannot be explained (at least not by a model with complete asset markets) by the large variations in real exchange rates which occured during the sample period.

While rejections of the implications of the complete asset markets model tested in this paper could be due to the fact that preferences or other aspects of the model are misspecified, the rejections cast strong doubt on the international Real Business Cycle model, as the preference specification and other features of the model tested in this paper are the ones commonly used in RBC models.

The findings presented in this section suggest that models with limitations on asset markets (see chapter III and Conze & Scheinkman(1991)) might be needed to gain a better understanding of the international covariation of consumption.

7. An Economy in Which Only Debt Contracts Can Be Used For International Borrowing and Lending

Chapter III of this thesis considers an economy with incomplete asset markets, in which only debt contracts can be used in international asset markets. As will be shown below, such a structure implies (in general) that consumption fails to be cointegrated in different countries.

Section 7.1. discusses testable implications for the model and it describes the statistical methods which will be used to test these implications. 7.2 presents the empirical results. Conclusion for section 7 are in 7.3.

7.1. *Testable Implications of the Debt Model*

7.1.1. A World With a Single Good

A one good world is considered first. The asset market structure is assumed to be the same as in chapter III and I initially assume a world with a single good. In period t, the only type of transaction between different countries consists in unconditional borrowing and lending at the real rate r_t using one period bonds: if country i makes a loan of A_t^i units of the consumption good in period t, then that country gets back $(1+r_t)*A_t^i$ units in period t+1.

Hence i's budget constraint in period t is:

$$c_t^i+I_t^i+A_t^i=(1+r_{t-1})*A_{t-1}^i+y_t^i, \tag{7.1}$$

where y_t^i is i's (gross) output in period t, while I_t^i is its net investment in physical capital between periods t and t+1.

The following constraint is imposed in order to rule out Ponzi schemes:

$$-Z\le A_t^i \quad \text{for all } t, \tag{7.2}$$

where Z is a large positive number.

Optimal behavior of country k implies that the following Euler condition is satisfied:

$$E_t\beta^{k}*(1+r_t)\{u_c^k(c_{t+1}^k)/u_c^k(c_t^k)\}=1 \ . \tag{7.3}$$

Hence expected intertemporal marginal rates of substitution are equated between countries:

$$\beta^{i}*E_t\{u_c^i(c_{t+1}^i)/u_c^i(c_t^i)\}=\beta^{j}*E_t\{u_c^j(c_{t+1}^j)/u_c^j(c_t^j)\} \quad \text{for } i\neq j. \tag{7.4}$$

For the iso-elastic utility function specified in (2.3), condition (7.4) can be restated as:

$$\beta^{i}*E_t\{(c_{t+1}^i/c_t^i)^{\sigma^i-1}\}=\beta^{j}*E_t\{(c_{t+1}^j/c_t^j)^{\sigma^j-1}\} \quad \text{for } i\neq j. \tag{7.4'}$$

In order to facilitate empirical testing of this condition, I follow Obstfeld (1989) and assume that consumption growth rates are jointly log-normally distributed and conditionally homoskedastic:[32] under the stated assumptions it follows from (7.4') that

$$(\sigma^i-1)*E_t\Delta\ln(c_{t+1}^i)=\mu^{i,j}+(\sigma^j-1)*E_t\Delta\ln(c_{t+1}^j) \ , \tag{7.5}$$

where $\mu^{i,j}$ is a constant.[33]

Hence we see that with log-normal and conditionally homoskedastic consumption growth and isoelastic utility functions, the condition that expected intertemporal rates of substitution are equated between countries implies that conditional expectations of growth rates of consumption between periods t and t+1 are perfectly correlated.

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[32]See also Hansen & Singleton (1983) for an example of the use of the assumptions of log-normality in the analysis of consumers' Euler equations.

[33]$\mu^{i,j}$ depends on the rates of time preference of countries i and j and on variances of consumption growth in these countries.

Note that condition (7.4) holds in any single good model in which there exists a riskless asset which can freely be traded by the residents of different countries. Hence it also holds with complete asset markets.

The key difference between incomplete and complete asset markets is that in the latter intertemporal marginal rates of substitution are equated ex post as well (and not merely in expected value), which implies that with isoelastic preferences, ex post growth rates of consumption are perfectly positively correlated between countries.[34] In general, consumption growth rates fail to be perfectly correlated between countries when asset markets are incomplete: it follows from (7.5) that

$$(\sigma^{i}-1)*\Delta\ln(c^{i}_{t+1})=\mu^{i,j}+(\sigma^{j}-1)*\Delta\ln(c^{j}_{t+1})+\eta_{t+1}, \qquad (7.5')$$

where $\eta_{t+1}\equiv(\sigma^{i}-1)*\varepsilon^{i}_{t+1}-(\sigma^{j}-1)*\varepsilon^{j}_{t+1}$, with $\varepsilon^{k}_{t+1}=\Delta\ln(c^{k}_{t+1})-E_{t}\Delta\ln(c^{k}_{t+1})$ for $k=i,j$. η_{t+1} is a linear combination of the forecast errors made in forecasting consumption growth between periods t and t+1 and therefore η_{t+1} is a serially uncorrelated random variable with mean zero.

Taking partial sums of (7.5') for periods $t=1,2,\ldots,T$, we get

$$(\sigma^{i}-1)*\ln(c^{i}_{T})=\vartheta+\mu^{i,j}*T+(\sigma^{j}-1)*\ln(c^{j}_{T})+H_{T}, \qquad (7.5'')$$

where $H_{T}\equiv\sum_{t=1}^{t=T}\eta_{t}$ and $\vartheta\equiv(\sigma^{i}-1)*\ln(c^{i}_{0})-(\sigma^{j}-1)*\ln(c^{j}_{0})$. In general, η_{T} is non-zero when asset markets are incomplete because nothing guarantees that in the absence of complete asset markets, unexpected consumption growth

[34] To see why this is so, note that taking first differences of (2.5) yields: $(\sigma^{i}-1)*\Delta\ln(c^{i}_{t})=\ln(\beta^{i}/\beta^{j})+(\sigma^{j}-1)*\Delta\ln(c^{j}_{t})$.

rates are perfectly correlated across countries [35] which implies that H_T follows a random walk and hence that log consumptions in countries i and j fail to be cointegrated.

7.1.2. **A World With One Country-Specific Good**

I next consider a model in which (as in section 2.2) each country consumes one country specific consumption good which is distinct from the consumption goods consumed by other countries. Assume that the residents of different countries can invest in risk-free bonds which are denominated in the country-specific goods of the different countries in the world. As in section 2, let $R^{i,j}$ denote the price of country i's good in terms of country j's good. Optimal behavior of countries i and j now implies that the following Euler conditions are satisfied:

$$E_t\beta^k*(1+r_t^k)*\{u_c^k(c_{t+1}^k)/u_c^k(c_t^k)\}=1 \quad \text{for } k=i,j \text{ and}$$
$$E_t\beta^h*(1+r_t^k)*\{[R_t^{k,h}/R_{t+1}^{k,h}]*u_c^h(c_{t+1}^h)/u_c^h(c_t^h)\}=1 \quad h,k:i,j;\ h\neq k, \tag{7.6}$$

where r_t^k is the real one period interest rate in terms of country k's good. It follows from (7.6) that for countries i and j:

$$E_t\beta^i*\{u_c^i(c_{t+1}^i)/u_c^i(c_t^i)\}=E_t\beta^j*\{[R_t^{i,j}/R_{t+1}^{i,j}]*u_c^j(c_{t+1}^j)/u_c^j(c_t^j)\}.$$

Assuming iso-elastic preferences, this condition implies that

$$E_t\beta^i*\{(c_{t+1}^i/c_t^i)^{\sigma^i-1}\}=E_t\beta^j*\{[R_t^{i,j}/R_{t+1}^{i,j}]*(c_{t+1}^j/c_t^j)^{\sigma^j-1}\}.$$

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[35] Note however that there exist special (and rather unlikely) circumstances where even with incomplete asset markets, unexpected consumption growth can be perfectly correlated between countries. This is for example the case if the outputs of all countries are perfectly correlated.

Assuming that rates of change of consumption and of real exchange rates are log-normally distributed and conditionally homoskedastic yields an expression of the following form:

$$(\sigma^{i}-1)*E_t\Delta\ln(c^{i}_{t+1})=\tilde{\mu}^{i,j}+E_t\Delta\ln(R^{i,j}_{t+1})+(\sigma^{j}-1)*E_t\Delta\ln(c^{j}_{t+1}), \tag{7.7}$$

The model with one country-specific consumption good therefore yields a linear restriction involving the conditional expectations of the rates of change of the consumptions of countries i and j and of their real exchange rate.[36]

The restrictions on the rates of change of consumptions and real exchange rates stated in (7.5) and (7.7) can compactly be expressed as:

$$\gamma'*X_{t+1}=\eta_{t+1}; \quad E_t\eta_{t+1}=0 \tag{7.8}$$

where γ is an nx1 vector of coefficients, while X_{t+1} is an nx1 vector containing the log growth rates of the consumptions and real exchange rates of countries i and j; for the model with a single consumption good, we have $X_{t+1}\equiv(\Delta\ln(c^{i}_{t+1}), \Delta\ln(c^{j}_{t+1}))'$ and $\gamma\equiv((\sigma^{i}-1), -(\sigma^{j}-1))'$; for the model with one country-specific consumption good, we have $X_{t+1}\equiv(\Delta\ln(c^{i}_{t+1}), \Delta\ln(c^{j}_{t+1}), \Delta\ln(R^{i,j}_{t+1}))'$ and $\gamma\equiv((\sigma^{i}-1), -(\sigma^{j}-1), -1)'$).

[36]Obstfeld (1989) derives (7.7) for a world in which *nominal* risk-free assets denominated in different national currencies are traded between the residents of these countries.

In what follows, Z_t denotes a kx1 (with k>n) vector of instruments which are contained in the period t information set and which therefore are orthogonal to the random variable η_{t+1}:

$$E_t\eta_{t+1}*Z_t=E_t\gamma'*X_{t+1}*Z_t=0 \tag{7.8'}$$

This orthogonality condition implies a restriction on the coefficients in linear regressions of the elements of the vector X_{t+1} on Z_t: consider the regression equation

$$X^m_{t+1}=\nu^m+\psi^{m,}*Z_t+\omega^m_{t+1}\ , \qquad \text{for } m=1,..,n \tag{7.9}$$

where X^m is the m^{th} element of the vector X and ψ^h is a kx1 vector of coefficients, while ν^m is an intercept.

The n equations stated in (7.9) can be written compactly as:

$$X_{t+1}=\nu+\Psi'*Z_t+\omega_{t+1}, \tag{7.10}$$

where $\nu\equiv(\nu^1,...,\nu^n)'$, $\omega_{t+1}\equiv(\omega^1_{t+1},..,\omega^n_{t+1})'$, while $\Psi\equiv(\psi^1,..,\psi^n)$ is a matrix of dimension kxn.

The orthogonality condition (7.8') implies the following restriction on the matrix Ψ:

$$\Psi*\gamma=0\ . \tag{7.11}$$

Hence the model of incomplete asset markets implies that the matrix Ψ is not of full rank:

$$\text{rank}(\Psi)<n. \tag{7.12}$$

A major difficulty in testing the model with incomplete asset markets is to find good instruments for future consumption growth. Our ability to accurately forecast future realizations of a variable depends on how strong the serial correlation of that variable is (at least when past realizations are used to make the forecasts). Table 4 reports the first 10 autocorrelations for non-durables and services consumption growth in the countries of the sample. While the serial correlation of consumption growth rates is relatively strong in Japan, it is quite weak for the other countries, which suggests that finding good instruments for consumption growth in these other countries might be difficult.

7.2 Test Results

The following tests of the model with incomplete asset markets are considered:[37]

(1.) The single good model implies that expectations of future consumption growth in two countries i and j conditional on a given set of instruments are perfectly correlated.

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[37] See Cumby & Huizinga (1991) for discussions of tests of condition (7.11) for the special case where n=2 (i.e. where the vector X consists of two elements). Alternatives to the tests considered in this paper can be found in recent work by Obstfeld (1989) and Barrionuevo (1991). Obstfeld jointly estimates the following condition (which follows from (7.7)) for a sample of countries which comprises the US, Japan and Germany, using three stage least squares:
$\Delta\ln(R^{i,j}_{t+1}) = -\tilde{\mu}^{i,j} + (\sigma^i - 1)*\Delta\ln(c^i_{t+1}) - (\sigma^j - 1)*\Delta\ln(c^j_{t+1}) + \tilde{\eta}^{i,j}_{t+1}$, where $\tilde{\eta}^{i,j}_{t+1}$ is a linear combination of the forecast errors made in forecasting the first difference of log consumptions and bilateral real exchange rates for countries i and j. Obstfeld argues that quarterly data for the period 1973:1-85:2 support (7.7).
Barrionuevo (1991) uses a Generalized Method of Moments framework to test the Euler conditions (7.6), using consumption and interest rate data for a sample of industrialized countries.

For each country pair i,j, table 11 reports sample correlation coefficents of fitted values of $\Delta\ln(c_t^i)$ and $\Delta\ln(c_t^j)$ which are obtained by regressing these variables on a constant and on lagged consumption growth rates in countries i and j. The table also reports cross-country correlations of actual consumption growth rates. For 11 of the 15 country pairs, the cross-country correlations of fitted consumption growth rates are larger than those of actual consumption growth rates. It appears however that the standard deviations of the cross-country correlations of fitted consumption growth rates are large: for 13 country pairs one fails to reject the hypothesis that the cross-country correlation of conditional expectations of future consumption growth is zero. This most likely reflects the low predictive power of the instruments used to predict future consumption growth in table 11.[38]

(2.) When the set of instruments used to test the orthogonality condition consists of lagged values of X_{t+1} (i.e. when $Z_t=(X'_t,X'_{t-1},\ldots,X'_{t-h})'$) a method due to Velu, Reinsel & Wichern (1986) [39] can be used to test the null-hypothesis that rank(Ψ)=n-1 against the alternative that rank(Ψ)=n.[40]

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[38]For each of the two regressions which are considered in table 11 for a given country pair, the table shows adjusted R^2s. The adjusted R^2s are quite low for many of the reported regressions.
The standard deviations for cross-country correlations of fitted consumption growth rates are calculated using the method presented in Cumby & Huizinga (1991).

[39]See Neusser (1991) for an interesting application of this test.

[40](7.2) requires that rank(Ψ)<n, not necessarily that rank(Ψ)=n-1. Velu et al. actually present n test statistics, which I will denote by

Table 13 presents p-values for tests of the hypothesis that rank(Ψ)=n-1 for the the single good model and for the model with one country-specific consumption good. These tests are conducted for h=0,1,2,3,4 , where h is the maximal lag of X_t used in the set of instruments.

For each model, table 13 therefore reports p-values for 75 Velu et al. test statistics.[41] The results cast doubt on the single good model: at the 10% level, the hypothesis rank(Ψ)=n-1 is rejected by 39 of the 75 Velu et al. test statistics; at the 20% level there are 54 rejections.

Note that if rank(Ψ)=n-1, then--in the single good model--there exists a unique value $(\sigma^i-1)/(\sigma^j-1)$ which satisfies the condition that $\Psi*\gamma=0$, where $\gamma\equiv(\sigma^i-1,\sigma^j-1)$. Velu et al. describe a method for obtaining a regression estimate $\hat{\Psi}$ of Ψ which satisfies the restriction that rank($\hat{\Psi}$)=n-1. Using $\hat{\Psi}*\gamma=0$ then allows to obtain an estimate of $(\sigma^i-1)/(\sigma^j-1)$. Table 12 reports estimates of $(\sigma^i-1)/(\sigma^j-1)$ which were obtained in this way. For the different values of the lag parameter 'h' considered in the table, the condition $(\sigma^i-1)/(\sigma^j-1)>0$ is typically rejected for more than two-thirds of the country pairs. It should however be noted that in most cases, the quantity $(\sigma^i-1)/(\sigma^j-1)$ is not estimated precisely; this most likely

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$v_0,\ldots,v_{n-1}$, where v_i tests the hypothesis that rank(Ψ)=i against the alternative that rank(Ψ)=n. For each country pair and value of h, I calculated these n test statistics. It appears that whenever v_j for j<n-1 rejects the hypothesis (at the 10% or the 20% level) that rank(Ψ)=j, then the statistic v_{n-1} rejects the hypothesis rank(Ψ)=n-1.

[41]Five values of 'h' are considered for 15 country pairs.

reflects the poor predictive power of lagged consumption growth for future consumption growth. In most of the cases where the estimate of $(\sigma^{i}-1)/(\sigma^{j}-1)$ is negative, we fail to reject (at conventional significance levels) the hypothesis that this quantity is positive.

Table 13 reports Velu et al. tests for the model with one country-specific good. The test results seem more favorable for the model than the test results obtained for the single good model: at the 10% level, the hypothesis rank(Ψ)=n-1 is rejected by 28 of the 75 Velu et al. test statistics; at the 20% level there are 34 rejections. It seems noteworthy, that most of the rejections (at the 10% level) of the model with one country-specific consumption good are accounted for by 5 country pairs: US-France, US-Italy, Japan-Italy, France-Italy and France-Canada. For the remaining country pairs there are only very few rejections of the restriction rank(Ψ)=n-1. It seems interesting that for the country pair US-Japan (the country pair which experienced the largest change in bilateral real exchange rates during the sample period), the condition rank(Ψ)=n-1 is rejected for the single good model, but not for the model with one country-specific consumption good.

Using an estimate $\hat{\Psi}$ of the matrix Ψ which satisfies the restriction that rank(Ψ)=n-1 allows to obtain estimates of the prefereence parameters σ^{i} and σ^{j} for a given country pair. For all country pairs in the sample, table 13 reports estimates of preference parameters which obtain for h=4. Most of the estimated preference parameters are consistent with concave utility functions.

(3.) Tests of the single good model were also conducted using the Generalized Method of Moments,[42] exploiting the orthogonality condition $E\gamma'*X_{t+1}*Z_t=0$ (see (7.8')). Table 14 presents results of these tests. For a given country pair i,j, the set of instruments used for the test consists of lagged growth rates of non-durables and services consumption and of lagged real interest rates in countries i and j (all instruments are lagged two and three periods).

Hansen's (1982) J-statistic shows that there is no country pair where the orthogonality condition (7.8') is rejected at the 10% significance level.[43]

7.3 *Summary*

This section has considered a model with incomplete asset markets in which only risk-free real debt contracts can be used for international borrowing and lending. In a setting with a single consumption good, this structure of asset markets implies (with iso-elastic preferences and log-normally distributed consumption growth rates) that conditional expectations of future growth rates of consumption are perfectly correlated across countries.

A setting is also considered in which each country consumes one country-specific consumption good. For this seeting, the structure of

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[42] See Hansen (1982) and Cumby, Huizinga & Obstfeld (1983).

[43] I experimented with other sets of instruments (including lagged rates of change of GNP and of real exchange rates). GMM failed to reject the single good model for these alternative sets of instruments.

incomplete asset markets implies a linear restriction on conditional expectations of future rates of change of consumption and real exchange rates.

GMM based tests of the single good model with incomplete asset markets fail to reject that model. Tests based on the Velu et a. (1986) method however suggest that the single good model is rejected.

Velu et al. tests of the model with one country-specific good show that it is strongly rejected for five of the 15 country pairs in the sample. There is hardly any evidence against the model for the remaining country pairs.