"Comment on 'A Forecasting Equation for the Canada-US Dollar Exchange Rate', by R. Amano and S. van Norden", in The Exchange Rate and the Economy, Proceedings of the 1992 Bank of Canada Conference; Bank of Canada, 1993, Ottawa, pp. 266-271.

Discussion

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I greatly enjoyed reading Robert Amano and Simon van Norden's study on the relationship between the Canada-U.S. bilateral real exchange rate (RPFX) and Canada's terms of trade during the 1971-91 period. The authors specifically focus on energy terms of trade (TOTENRGY) and on commodities terms of trade (TOTCOMOD). They argue that RPFX, TOTENRGY and TOTCOMOD (in logs) follow unit-root processes and that these variables are cointegrated — i.e., that there is a linear restriction that ties together the trend behaviour of these variables. The paper shows that out-of-sample forecasts of RPFX, which are generated on the basis of past values of RPFX, TOTENRGY, TOTCOMOD and RDIFF, outperform forecasts generated using a random-walk model of the real exchange rate. This is an interesting result, given the finding by Meese and Rogoff (1983) that a random-walk model of the exchange rate outperforms several structural models of the exchange rate.

In my comments, I will focus on the evidence concerning unit roots and the issue of cointegration between the real exchange rate and the terms of trade. The authors kindly made available to me the data used in their study.

Applying the Augmented Dickey-Fuller (ADF) (1979) unit-root test, I find no evidence against the unit-root hypothesis for the logarithms of RPFX and TOTENRGY, which confirms the analysis of Amano and van Norden. However, the

* *I thank R. Amano and S. van Norden for useful discussions.*

evidence concerning a unit root in ln(TOTCOMOD) is inconclusive. Table 1 reports results from the ADF test. We see that the outcome of the ADF test depends critically on the lag length used in the test: for low lag lengths, the unit-root hypothesis is not rejected, while for lag lengths greater than 6, the null hypothesis of a unit root in the log of TOTCOMOD is rejected at 10 per cent.

Table 1

ADF Tests for Unit Roots in Log of Non-Energy Commodity Terms of Trade

Lag length	1	2	3	4	5	6	7	8
Test statistic	-2.26	-2.08	-2.12	-2.42	-2.68	-2.80	-3.34	-3.25
							(*)	(*)

Lag length	9	10	11	12	13	14	15
Test statistic	-3.26	-3.73	-3.91	-3.66	-4.05	-4.28	-3.91
	(*)	(**)	(**)	(**)	(***)	(***)	(**)

Note: (*), (**) and (***) denote rejections of the null hypothesis that the series has a unit root at the 10 per cent, 5 per cent and 1 per cent levels, respectively. The table allows for a linear deterministic trend in the process for ln(TOTCOMOD). Under the null hypothesis $\ln(TOTCOMOD) = a + b^{*}t + Z_t$ holds, where $\{Z_t\}$ has mean zero. If $\{Z_t\}$ follows an $AR(k+1)$ process with a unit root, then $\phi = 0$ holds in the following model: $\Delta x_t = \alpha + \beta^{*}t + \phi^{*}x_{t-1} + \sum_{i=1}^{k}\Psi_i{}^{*}\Delta x_{t-i} + \varepsilon_t$. The table reports studentized values of the OLS estimates of ϕ for different choices of the lag length "k." Critical values are tabulated in Fuller (1976).

I applied the $\hat{Z}_\alpha$ and $\hat{Z}_t$ single-equation cointegration tests described by Phillips and Ouliaris (1990) to data on RPFX, TOTENRGY and TOTCOMOD. Test results are shown in Table 2. According to these results, one cannot reject the null hypothesis of no cointegration, even when a 50 per cent significance level is used!

In contrast, the single-equation cointegration tests presented by Amano and van Norden yield results that are much more consistent with cointegration (see Tables 3 and 4 of their paper).

Table 2

Phillips and Ouliaris (1988) Tests of No-Cointegration Hypothesis

Left-hand-side variable	$\hat{Z}_{\alpha}$	$\hat{Z}_{t}$
ln(RPFX)	-8.00	-1.95
ln(TOTCOMOD)	-13.25	-2.68
ln(TOTENRGY)	-8.41	-2.10

Note: Test results are reported for all possible choices for the left-hand-side variable used in the cointegrating regression. The first column indicates what variable is used on the left-hand side of the cointegrating regression. "ln" refers to the natural logarithm.

To compute the test statistics, it is necessary to correct for serial correlation in the first differences of the residuals of the cointegrating regressions. The Newey and West (1987) method was used for that purpose, allowing for 10 autocorrelations (the results are not sensitive to the selected number of autocorrelations). A linear time trend was included in the cointegrating regression. The critical values for the $\hat{Z}_{\alpha}$ and $\hat{Z}_{t}$ test statistics are -15.89 and -2.85, respectively, at the 50 per cent level (the null hypothesis is rejected if the test statistic is smaller than the critical value). These critical values were obtained using a Monte Carlo experiment with 2,000 replications of trivariate random-walk time series (of length 500) with independent Gaussian innovations.

Amano and van Norden also apply Johansen's (1988) multi-equation test for cointegration. This test assumes that the variables under consideration follow a vector autoregression (VAR) with Gaussian errors. While experimenting with the Johansen test, I found that the choice of the order of the VAR is critical for the outcome of the test. The Schwarz criterion (see Judge et al., 1985), selects a VAR of order 1 when applied to the system of variables consisting of ln(RPFX), ln(TOTCOMOD) and ln(TOTENRGY). In contrast, Amano and van Norden work with a VAR of order 20 (they use a likelihood-ratio test suggested by Doan (1990) to determine the lag length). Table 3 shows results from Johansen tests that are based on a VAR of order 2. Using a 5 per cent significance level, we conclude from

Table 3 that there are two linearly independent cointegrating vectors in the system of variables consisting of ln(RPFX), ln(TOTENRGY) and ln(TOTCOMOD), and not just one cointegrating vector, as suggested by Amano and van Norden.

Applying the Johansen test to the bivariate system consisting of ln(RPFX) and ln(TOTENRGY), as well as to the system consisting of ln(RPFX) and ln(TOTCOMOD), yields no evidence of cointegration. It appears, however, that ln(TOTENRGY) and ln(TOTCOMOD) are cointegrated, as can be seen from Table 4. In fact, Table 4 suggests (at the 1 per cent level) that there are two cointegrating vectors in the system consisting of ln(TOTENRGY) and ln(TOTCOMOD), which can only be true if both variables are trend-stationary (recall that the ADF unit-root tests presented in Table 1 are not inconsistent with the hypothesis that ln(TOTCOMOD) is trend-stationary). This suggests that the two cointegrating relations in the trivariate system, ln(RPFX), ln(TOTENRGY) and ln(TOTCOMOD), merely reflect the fact that the last two variables are trend-stationary.

Table 3
Johansen (1988) Tests for Cointegration: Variables in System — 1n(RPFX), 1n(TOTENRGY), 1n(TOTCOMOD)

Number of cointegrating vectors	Trace statistic	Maximum-likelihood statistic
< 3	47.89 (***)	29.28 (***)
< 2	18.60 (**)	15.46 (**)
< 1	3.14	3.14

Note: The test statistics reported in this table are based on a VAR of order 2. Asterisks (**) and (***) denote significance at 5 per cent and 1 per cent levels, respectively. (Critical values from Table A2 of Johansen and Juselius, 1990.)

\In conclusion, the test results described in these comments differ from those presented by Amano and van Norden. The fact that different statistical methods yield widely varying results concerning unit roots and cointegrating relations most likely reflects the shortness of the sample period considered in the paper (approximately 20 years of data). Hence, it would be an interesting task for future research to investigate the relationship between Canada's real exchange rate and the terms of trade using a longer sample period.

Table 4

Johansen (1988) Tests for Cointegration: Variables in System — 1n(RPFX), 1n(TOTENRGY), 1n(TOTCOMOD)

Number of cointegrating vectors	Trace statistic	Maximum-likelihood statistic
< 2	33.34 (***)	19.94 (***)
< 1	13.40 (***)	13.40 (***)

Note: See note to Table 3.

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