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Estimating the state vector of linearized DSGE models without the Kalman filter

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HIGHLIGHTS

- I present a simple method for estimating the state vector of linearized DSGE models.
- The method does not use the Kalman filter.
- It can easily handle filtered data and arbitrary patterns of missing observations.
- A formula for the conditional covariance matrix of the state vector is provided.

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ABSTRACT

This note presents a *simple* method for estimating the state vector of linearized DSGE models without using the Kalman filter. The conditional covariance matrix of the state vector is also derived. The method can easily cope with filtered data, and with arbitrary patterns of missing observations.

This note presents a *simple* method for estimating the state vector of linearized dynamic stochastic general equilibrium (DSGE) models, without using the Kalman filter. The method can easily handle filtered data and arbitrary patterns of missing observations.

The solution of generic linearized DSGE models can be represented as

$$x_t = h \cdot x_{t-1} + \eta \cdot \varepsilon_t,$$

where x_t is an $n_x \times 1$ vector of (possibly unobserved) endogenous and exogenous state variables, while ε_t is a white noise vector that is orthogonal to x_s for $s < t$. h and η are matrices whose coefficients are functions of structural model parameters; see, e.g., Schmitt-Grohé and Uribe (2004, 2011). Assume that x_t is stationary, and that ε_t is normally distributed.

Let $X = [x_1; x_2; \cdots; x_T]$ denote the column vector obtained by stacking the state vectors for periods $t = 1, \ldots, T$. Date t observables are given by the $n_y \times 1$ vector $y_t \equiv g_t \cdot X$, with $n_y < n_x$; g_t is a $n_y \times (n_x \cdot T)$ matrix of deterministic (possibly time-varying) coefficients; thus y_t is a subset (or linear combination) of the state vector X. This specification encompasses cases in which observables y_t just depend on the date t state vector, $y_t \equiv g \cdot x_t$, but also captures cases in which the data is filtered using one-sided or two-sided filters (such as the HP filter) so that y_t also depends on x_s for $s \neq t$. Let $Y \equiv [y_1; y_2; \cdots; y_T]$ denote the stacked vector of observables for $t = 1, \ldots, T$.

This note shows how to estimate the (stacked) state vector X, given data Y, very simply, without using the Kalman filter. Estimates of (latent) states are often of great interest for economic analysis.

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The method is inspired by Schmitt-Grohé and Uribe (2011) who show how to construct the likelihood of Y without using the Kalman filter, by interpreting Y as a draw from a multivariate density.[1]

Observe that the 'smoothed' estimate of X, i.e. the conditional expectation of X given Y, is

$$E[X|Y] = E(XY')\{E(YY')\}^{-1} \cdot Y,$$

due to the joint normality of X, Y.

Note that $Y = G \cdot X$, where $G \equiv [g_1; g_2; \cdots; g_T]$. Thus, $E(YY')$ and $E(XY')$ can be computed as follows: $E(YY') = GE(XX')G'$, $E(XY') = E(XX')G'$, with

$$E(XX') = \begin{bmatrix} Ex_1x_1{}' & \cdots & Ex_1x_T{}' \\ \cdots & \cdots & \cdots \\ Ex_Tx_1{}' & \cdots & Ex_Tx_T{}' \end{bmatrix},$$

where $Ex_sx_t{}' = (h)^{s-t}\Sigma_x$ for $s > t$ and $Ex_sx_t{}' = \Sigma_x(h')^{t-s}$ for $s \leq t$, with $\Sigma_x \equiv E(x_tx_t{}')$.

The smoothed estimate of the vector of innovations ε_t can be computed using

$$E[\varepsilon_t|Y] = (\eta' \cdot \eta)^{-1} \cdot \eta' \cdot (E[x_t|Y] - h \cdot E[x_{t-1}|Y]).$$

The conditional covariance matrix of X (useful for computing confidence sets for the state variables) is

$$E[XX'|Y] = E(XX') - E(XY')\{E(YY')\}^{-1}E(YX').$$

Estimates of the state vector X based on smaller data sets (e.g. due to missing observations) can be computed by adapting the above formulae. Let $Y^A = W \cdot Y$ be a vector consisting of a subset of the elements of Y, where W is a $n_W \times (n_Y \cdot T)$ 'selection' matrix of zeros and ones with $n_W < n_Y \cdot T$. Then

$$E[X|Y^A] = E(XY')W'\{WE(YY')W'\}^{-1} \cdot Y^A$$

and

$$E[XX'|Y^A] = E(XX') - E(XY')W'\{WE(YY')W'\}^{-1}WE(YX').$$

Computer code that implements the algorithm is available on the author's web page.

References

Schmitt-Grohé, S., Uribe, M., 2004. Solving dynamic general equilibrium models using a second-order approximation to the policy function. Journal of Economic Dynamics and Control 28, 755–775.

Schmitt-Grohé, S., Uribe, M., 2011. Evaluating the sample likelihood of linearized DSGE models without the use of the Kalman filter. Economics Letters 109, 142–143.

[1] As pointed out by Schmitt-Grohé and Uribe (2011), the (recursive) Kalman filter cannot be employed when the data are filtered using a two-sided filter.