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The real exchange rate and household consumption heterogeneity: Testing Kocherlakota and Pistaferri's (2007) model

Robert Kollmann *

Solvay Brussels School of Economics & Management, Université Libre de Bruxelles, Brussels, Belgium
Centre for Economic Policy Research (CEPR), London, UK
ERUDITE, Université Paris-Est Créteil (UPEC), Créteil, France

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ABSTRACT

Does household heterogeneity matter for exchange rate determination? This paper tests Kocherlakota and Pistaferri's (2007) prominent heterogeneous agent model, in which the real exchange rate *perfectly* tracks relative domestic/foreign moments of cross-household consumption distributions. The evidence presented here indicates that the real exchange rate is disconnected from relative cross-household consumption moments.

1. Introduction

Widely used models of the world economy postulate complete financial markets and efficient domestic and international risk sharing (see, e.g., the canonical International RBC model of Backus et al., 1994). With full risk sharing, the ratio of domestic to foreign representative households' marginal utilities of *aggregate* consumption is proportional to the real exchange rate. Yet, empirically, real exchange rates are uncorrelated with relative (domestic/foreign) aggregate consumption (Kollmann, 1991, 1995; Backus and Smith, 1993). That "consumption-real exchange rate anomaly" is one of the major puzzles in international macroeconomics (Itskhoki and Mukhin, 2021).

This raises the question whether models with financial frictions and household heterogeneity can better account for the actual dynamics of consumption and the real exchange rate. Kocherlakota and Pistaferri (2007) [KP] developed a prominent model of a two-country world with restricted risk sharing, as household-specific productivity shocks are assumed to be private information; the resulting equilibrium allocations are Pareto optimal, subject to incentive compatibility constraints implied by private information. This "private-information Pareto-optimal" (PIPO) structure entails that the real exchange rate *perfectly* tracks the domestic/foreign ratio of γ-th non-central moments of cross-household consumption distributions, where γ is the risk aversion coefficient. Thus, household heterogeneity is predicted to be a key determinant of the real exchange rate; for $\gamma>1$ the real exchange rate is especially sensitive to the consumption of the rich.

KP test this prediction with US and UK household-level consumption data. Using regression tests, the authors conclude that, for $\gamma\approx5$, the UK/US ratio of γ-th cross-household consumption moments "tracks the real exchange rate well" (p.C17).

This paper assesses KP's empirical evidence, and provides new tests, using KP's data.[1] I document that relative domestic/foreign sample moments of cross-household consumption distributions are much more volatile than the real exchange rate, and uncorrelated with the real exchange rate. The model error is correlated with relative UK/US industrial production and stock prices, as well as with future values of the real exchange rate. These results suggest that household heterogeneity, of the type highlighted by the PIPO model, fails to account for the dynamics of the UK/US real exchange rate.

2. Consumption and the real exchange rate: theory

Under CRRA utility with risk aversion γ, a representative agent model with full risk sharing implies:

$$\ln e_t^{j,k} = \gamma \ln(C_t^j/C_t^k) + \upsilon^{j,k}, \tag{1}$$

* Correspondence to: SBS-EM, ULB, 50 Av.Roosevelt, 1050 Brussels, Belgium.
E-mail address: robert_kollmann@yahoo.com.

[1] I thank KP for sharing their data.

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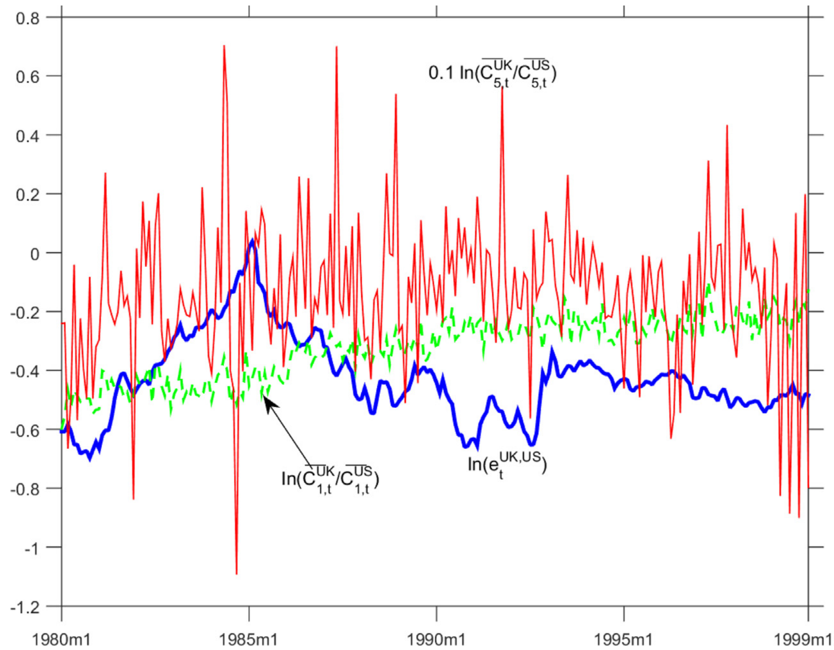

Fig. 1. The Figure shows monthly time series (1980–1999) of the logged UK/US real exchange rate $\ln(e_t^{UK,US})$, and of relative UK/US cross-household consumption sample moments of orders $\gamma = 1$ $[\ln(\overline{C_{1,t}^{UK}}/\overline{C_{1,t}^{US}})]$ and $\gamma = 5$ $[0.1 \times \ln(\overline{C_{5,t}^{UK}}/\overline{C_{5,t}^{US}})]$. **Note: plotted** $\ln(\overline{C_{5,t}^{UK}}/\overline{C_{5,t}^{US}})$ **is adjusted by factor 0.1**. For interpretation of the references to color in this Figure, the reader is referred to the web version of the article.

Table 1
Properties of relative cross-household consumption moments of order γ.

	γ								
	1	2	3	4	5	6	7	8	9
Std. of $\ln(\overline{C_{\gamma,t}^{UK}}/\overline{C_{\gamma,t}^{US}})$	10.3%	29.7%	88.5%	166.8%	245.4%	320.2%	391.8%	461.1%	528.9%
Autocorrel. of $\ln(\overline{C_{\gamma,t}^{UK}}/\overline{C_{\gamma,t}^{US}})$	0.84	0.43	0.14	0.09	0.08	0.07	0.07	0.07	0.07
$Corr(\ln(\frac{\overline{C_{\gamma,t}^{UK}}}{\overline{C_{\gamma,t}^{US}}}), \ln(e_t^{UK,US}))$	-0.31	-0.17	-0.02	0.04	0.06	0.06	0.06	0.06	0.06

Note—The Table reports standard deviations and autocorrelations of monthly series (1980–1999) of the logged relative UK/US cross-household γth consumption sample moments ($\gamma = 1, 2, \ldots, 9$) as well as correlations with the logged real exchange rate.

where $e_t^{j,k}$ is the date t real exchange rate between countries j and k, defined as the ratio of k's CPI to j's CPI (in same currency). C_t^j $[C_t^k]$ is aggregate consumption in country $j[k]$. $\upsilon^{j,k}$ is a fixed term reflecting countries' relative initial wealth. (1) implies that the real exchange rate perfectly tracks relative aggregate consumption. As discussed above, this prediction is rejected empirically.

By contrast, KP's heterogeneous agent model (PIPO) implies that the real exchange rate perfectly tracks relative γ-th cross-household consumption moments:

$$\ln e_t^{j,k} = \ln(C_{\gamma,t}^j / C_{\gamma,t}^k) + \upsilon_\gamma^{j,k}, \quad (2)$$

where $C_{\gamma,t}^j$ $[C_{\gamma,t}^k]$ is the γ-th non-central *population* moment of the cross-household consumption distribution in country $j[k]$, at t; $\upsilon_\gamma^{j,k}$ is a constant.

3. Testing the heterogeneous agent model

KP test (2) with monthly US and UK household consumption data (from the CEX and FES surveys), 1980–1999. Let $\overline{C_{\gamma,t}^j}$ be the γ − th non-central **sample** moment of the consumption distribution in country j, at t.

Table 1 reports the standard deviation and autocorrelation of the logged monthly series $\ln(\overline{C_{\gamma,t}^j}/\overline{C_{\gamma,t}^k})$ (j = UK, k = US), as well as its correlation with the logged real exchange rate $\ln(e_t^{j,k})$, for $\gamma = 1, 2, \ldots, 9$. Fig. 1 plots $\ln(e_t^{j,k})$ [thick solid blue line] as well as relative cross-household consumption moments for $\gamma = 1$ and $\gamma = 5$: $\ln(\overline{C_{1,t}^j}/\overline{C_{1,t}^k})$ [dashed green line] and $0.1 \times \ln(\overline{C_{5,t}^j}/\overline{C_{5,t}^k})$ [thin solid red line]. (Note scaling of plotted moments for $\gamma = 5$.)

These statistics and plots were *not* provided in KP's paper; they suggest that the real exchange rate is disconnected from relative cross-household consumption moments.

The standard deviation and autocorrelation of the monthly real exchange rate are 13.9% and 0.99, respectively. For $\gamma \geq 2$, the relative cross-household consumption moments are several times more volatile than the real exchange rate. Relative consumption moments are much less persistent than the real exchange rate, and negatively or weakly positively correlated with the real exchange rate. The "model error" $\ln e_t^{j,k} - \upsilon_\gamma^{j,k} - \ln(\overline{C_{\gamma,t}^j}/\overline{C_{\gamma,t}^k})$ is about as volatile as $\ln(\overline{C_{\gamma,t}^{UK}}/\overline{C_{\gamma,t}^{US}})$.

KP argue that for $\gamma \approx 5$ the heterogeneous agent model fits the real exchange rate. Yet, for $\gamma = 5$, the relative consumption moment is 17.6 times (!) more volatile than the real exchange rate, and uncorrelated with the real exchange rate (see Figure and Table 1).

This casts doubts on the heterogeneous agent model. However, KP argue that the high volatility of $\ln(\overline{C_{\gamma,t}^{UK}}/\overline{C_{\gamma,t}^{US}})$ might reflect sampling error, and they thus use an "indirect" statistical test of (2).

Table 2
Slope estimates in regressions of model error on selected variables.

γ	Quarterly 1st differences	Annual 1st differences	Levels
(1)	*(2)*	*(3)*	*(4)*
(a) Regressions on real exchange rate			
1	**1.04** (0.08)	**1.04** (0.05)	**1.23** (0.04)
2	**1.34** (0.45)	**1.16** (0.21)	**1.38** (0.13)
3	1.67 (1.55)	**1.30** (0.71)	**1.12** (0.41)
4	1.29 (2.97)	1.17 (1.36)	0.54 (0.77)
5	0.45 (4.29)	0.93 (2.00)	-0.02 (1.14)
6	-0.52 (5.60)	0.69 (2.61)	-0.47 (1.49)
7	-1.48 (6.87)	0.48 (3.20)	-0.85 (1.82)
8	-2.39 (8.09)	0.29 (3.76)	-1.19 (2.15)
9	-3.23 (9.30)	0.12 (4.31)	-1.51 (2.46)
(b) Regressions on relative industrial production			
1	0.07 (0.33)	0.14 (0.51)	-0.18 (0.25)
2	0.24 (1.23)	0.39 (0.80)	**-1.29** (0.44)
3	-0.58 (4.12)	0.63 (2.09)	**-3.85** (1.13)
4	-4.78 (7.73)	-0.20 (3.88)	**-6.94** (2.10)
5	-10.64 (11.35)	-1.81 (5.70)	**-9.92** (3.09)
6	-16.64 (14.81)	-3.58 (7.43)	**-12.71** (4.04)
7	-22.32 (18.14)	-5.29 (9.05)	**-15.37** (4.95)
8	**-27.67** (21.37)	-6.88 (10.69)	**-17.95** (5.83)
9	**-32.74** (24.54)	-8.39 (12.26)	**-20.48** (6.68)
(c) Regressions on relative stock price			
1	**-0.39** (0.08)	**-0.64** (0.10)	**-0.97** (0.04)
2	**-0.86** (0.31)	**-0.91** (0.19)	**-1.46** (0.10)
3	**-1.88** (1.07)	**-1.48** (0.60)	**-1.90** (0.33)
4	**-2.76** (2.03)	**-1.88** (1.17)	**-2.12** (0.65)
5	-3.40 (2.99)	-2.14 (1.74)	**-2.26** (0.96)
6	-3.93 (3.91)	-2.38 (2.28)	**-2.42** (1.26)
7	-4.40 (4.80)	-2.62 (2.81)	**-2.62** (1.55)
8	-4.87 (5.65)	-2.86 (3.31)	**-2.38** (1.82)
9	-5.33 (6.49)	-3.11 (3.80)	**-3.05** (2.10)

Note—The Table reports slope coefficients in regressions of model error on the logged real exchange rate (Panel (a)); on logged relative industrial production (Panel (b)); and on the logged relative stock price (Panel (c)). In parentheses: Newey–West standard errors (number of Newey–West lags: three, twelve and zero in Cols.(2),(3) and (4), respectively; std.errors are not sensitive to number of lags).
Coefficients underlined by a continuous [dotted] line are significant at 5% [10%] level (one-sided test).
Industrial production (IP): from IFS; relative IP has downward trend. I thus use linearly detrended logged relative IP as regressor. Stock prices are cumulated dollar stock returns (from K.French database).

The test exploits the fact that, when (2) is true, the model error solely reflects *sampling* error of cross-household moments. One can thus test (2) by regressing the model error on any variable that is (plausibly) uncorrelated with consumption sampling error. The slope coefficient of that regression should be close to zero, and statistically insignificant. KP apply this idea by regressing the "differenced" model error on the differenced logged real exchange rate:

$$\Delta_u\{\ln e_t^{j,k} - \ln(\overline{C^j_{\gamma,t}}/\overline{C^k_{\gamma,t}})\} = b\Delta_u \ln e_t^{j,k} + \eta_t, \quad (3)$$

where $\Delta_u x_t \equiv x_t - x_{t-u}$; η_t is a regression error. (2) implies $b = 0$. KP **only** consider $u = 3$ (monthly observations of **quarterly** 1st differences). KP do *not* consider regressors other than the real exchange rate.

In Panel (a) of Table 2, Column (2) reports estimates of b for $u = 3$ and $\gamma = 1, \ldots, 9$. The slope estimate is zero for $\gamma = 5.47$; for smaller [larger] values of γ, the estimates of b are positive [negative]. This is the basis of KP's claim that the heterogeneous agent model "is able to account for movements in the real exchange rate" for $\gamma \approx 5$ (p.C3). Importantly, the estimates of b in Panel (a) are not statistically significant, when $\gamma > 2$. For $\gamma = 5$ one cannot reject the hypothesis that b equals any value between −7 and +7. It is thus important to investigate the robustness of KP's regression-based test results.

Table 3
p-values of first 12 leads of real exchange rate.

γ	Quarterly 1st differences	Annual 1st differences	Levels
(1)	(2)	(3)	(4)
1	0.43	0.00	0.00
2	0.09	0.01	0.00
3	0.06	0.01	0.07
4	0.04	0.00	0.07
5	0.02	0.00	0.06
6	0.02	0.00	0.04
7	0.01	0.00	0.04
8	0.01	0.00	0.03
9	0.01	0.00	0.03

Note—This Table is based on regressions of model error on the current logged real exchange rate, and on the first 12 lags and leads of the logged real exchange rate. The regressions are run in quarterly 1st differences, annual 1st differences and levels. The Table reports p-values of Wald tests that the 12 leads of the logged real exchange rate all have zero coefficients.

3.1. Regressions based on annual 1st differences and levels

In Panel (a) of Table 2, Col. (3) reports slope coefficients based on regression (3) with $u = 12$ (monthly series of **annual** 1st differences), while Col. (4) considers regressions in **levels**:

$$\ln e_t^{j,k} - \ln(\overline{C^j_{\gamma,t}}/\overline{C^k_{\gamma,t}}) = a + b \ln e_t^{j,k} + \eta_t. \quad (4)$$

The 'levels' regressions (4) yield results in line with KP's results. By contrast, the 'annual 1st differences' regressions overturn KP's findings, in the sense that the slope coefficient is positive for *all* considered values of γ.[2] However the slope b is again estimated imprecisely for large γ. I investigate next whether other regressors yield more precisely estimated slope coefficients.

3.2. Other regressors

I added the first 12 **lags and leads of the logged UK/US real exchange rate** as regressors to Eqs. (3) (with $u = 3, 12$) and (4). *Lagged* exchange rates are significant in the 'annual 1st differences' regressions. Model error is very significantly correlated with the **future** real exchange rate, for all γ; see Table 3 which reports p-values close to zero for tests of the hypothesis that all leads of the exchange rate have zero coefficients.

Panels (b) and (c) of Table 2 regress model error on logged **relative UK/US industrial production and on stock prices**. 'Quarterly' and 'annual' 1st differences regressions (Cols.(2),(3)) on the relative stock price produce slope coefficients that are negative for all γ, and often highly significant. In the 'levels' regressions (Col.(4)), both relative industrial production and the relative stock price have large negative slope coefficients that are highly statistically significant, for all γ.

2 The slope coefficient is negative for $\gamma > 10$, but such large values of γ are outside the range generally considered in macroeconomic models. Standard deviations of model error (>600%) are enormous for $\gamma > 10$.

4. Conclusion

The findings reported here imply a clear rejection of the heterogeneous-agent theory's real exchange rate equation. The link between the real exchange rate and consumption (heterogeneity) remains a puzzle.

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