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A tractable overlapping generations structure for quantitative DSGE models

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ABSTRACT

This paper develops a novel tractable overlapping generations (OLG) structure whose aggregate equations resemble a model of an infinitely lived representative agent, except that there are no aggregate transversality conditions in the OLG economy. The main assumptions are complete markets and time-invariant (but age-dependent) consumption shares of age-groups. The tractability of the OLG structure here distinguishes it from conventional OLG models — the present structure is suitable for quantitative dynamic stochastic general equilibrium (DSGE) macro models. Importantly, the OLG structure here maintains key predictions of standard OLG models, namely the possibility of low real interest rates and of equilibrium indeterminacy.

1. Introduction

This paper develops a novel tractable overlapping generations (OLG) structure whose aggregate equations resemble a model of an infinitely lived representative agent (RA), except that there are no aggregate transversality conditions (TVC) in the OLG economy. Specifically, the OLG structure here yields a (quasi-) Euler equation in terms of *aggregate* (economy-wide) consumption that is isomorphic to the conventional RA Euler equation. However, the subjective discount factor in the aggregate OLG Euler equation can differ from the subjective discount factor in RA models — the aggregate OLG discount factor may exceed unity. The format of other aggregate equations remains unchanged. Importantly, the structure here maintains key predictions of standard OLG models, namely the possibility of low (even negative) real interest rates and of equilibrium indeterminacy.

In essence, this paper shows that one can transform an RA dynamic stochastic general equilibrium (DSGE) model into an OLG-DSGE model (with the OLG structure presented here) just by dropping the RA's TVC, but the OLG structure allows greater freedom in calibrating the subjective discount factor. The resulting OLG-DSGE model can be solved as conveniently as the RA-DSGE model.

The central assumption of the proposed OLG structure is that, at each date, newborn agents receive a wealth transfer (from older agents) that is set such that equilibrium consumption during the first period of life represents a *time-invariant* fraction of aggregate consumption. If consumption risk is efficiently shared among all *contemporaneous* age-groups, via complete insurance markets (as often postulated in DSGE models), then the assumed time-invariant consumption share of newborn agents implies that (under CRRA utility) *each* older age-group's equilibrium consumption likewise represents a time-invariant, but age-dependent, share of aggregate consumption. This permits the aggregation of Euler equations across age-groups. That aggregation result holds irrespective of agents' life-spans; thus, the present OLG framework can easily handle agents with long life-spans.

To justify the central assumption (about transfers), it can be noted that, in reality, there are large transfers to young individuals, driven by altruism and social norms (e.g., parental support/bequests; tax-funded child benefits and education spending). Empirically, the per capita consumption of younger consumers closely tracks economy-wide per capita consumption; changes in the *relative* consumption of different age-groups are dwarfed by changes in the aggregate consumption *level* (e.g., Saito, 2001). This motivates the assumption here that the consumption share of newborn agents is time-invariant. In the present OLG structure, the transfers to newborns, and the time-invariant newborn consumption share, are taken as an exogenous institutional/sociopolitical datum that is not modeled explicitly.

The tractability of the present OLG structure constitutes an important advantage over standard OLG models, especially when agents are long-lived. OLG models are workhorses of dynamic

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macroeconomic theory, and they represent a key alternative to RA models. In OLG models, competitive equilibria may fail to be Pareto-optimal; OLG models may have multiple equilibria and imply low interest rates.[1] For analytical convenience, much of the *theoretical* OLG literature assumes that individuals live only two (or three) periods. Such a setting is not suitable for studying quarterly or annual economic fluctuations. A quarterly OLG business cycle model with realistic life expectancy would need to assume agents with a life-span of several hundred periods. Solving *stochastic* models with a large number of overlapping *heterogeneous* age cohorts is challenging. This explains why the overwhelming majority of quantitative DSGE macro models (see survey by Wieland et al., 2016) does not use OLG structures, but assumes an RA setting. By contrast, the time-invariance of age-specific consumption shares, in the novel OLG structure here, makes solving quantitative OLG-DSGE models simpler. A drawback of the present OLG structure (which it has in common with RA models) is that it is not well suited to address distributional concerns, both within as well as across generations, due to the assumed efficient risk sharing (in the OLG structure here, all members of the same generation have identical consumption; the structure allows for intergenerational consumption differences, but age-specific consumption shares are time-invariant, as mentioned above). Nevertheless, the structure here is useful as it generates key predictions of standard OLG models, while only making minimal changes to an RA setup.

2. A tractable OLG structure

Assume an economy in which a measure 1 of agents is born each period. All agents live $N < \infty$ periods, and thus the total population is N. Agents who are in the ith period of their life at date t are referred to as 'age-group i' at date t. All members of the same age-group are identical. Let $c_{i,t}$ denote the consumption of age-group i at t. The expected life-time utility of the generation born at t is

$$E_t \sum_{s=1}^{N} \beta^{s-1} \log(c_{s,t+s-1}), \tag{1}$$

where $\beta > 0$ is the subjective discount factor. (For simplicity, the presentation here abstracts from labor/leisure choices. The key results continue to hold under non-unitary consumption risk aversion, and when the subjective discount factor is age dependent.) $\beta < 1$ is not required here, as life-time utility is bounded even if $\beta \geq 1$, given the finite life span N. By contrast, $\beta < 1$ has to hold for an infinitely-lived agent. Aggregate consumption is $C_t \equiv \sum_{i=1}^{N} c_{i,t}$.

2.1. Risk sharing

Assume that, at each date t, a complete set of one-period Arrow–Debreu insurance contracts is traded, so that, in equilibrium, consumption risk is efficiently shared across all agents alive at both t and $t+1$. Under the assumed CRRA (log) period utility, this implies that consumption growth between t and $t+1$ is equated across those contemporaneous agents, for all states of the world:

$$c_{i+1,t+1}/c_{i,t} = c_{2,t+1}/c_{1,t} \quad \text{for} \quad i = 2, \ldots, N-1. \tag{2}$$

Let $\lambda_{i,t} \equiv c_{i,t}/C_t \geq 0$ denote the share of age-group i's consumption in aggregate consumption, at t.

(2) implies

$$\lambda_{i+1,t+1}/\lambda_{i,t} = \lambda_{2,t+1}/\lambda_{1,t} \quad \text{for} \quad i = 2, \ldots, N-1. \tag{3}$$

(3) and the adding up constraint

$$\sum_{i=1}^{N} \lambda_{i,t+1} = 1 \tag{4}$$

define $N-1$ equations in the N consumption shares at $t+1$. Assume that the equilibrium consumption share of age-group 1 is time-invariant: $\lambda_{1,t} = \lambda_1 \; \forall t$, which requires a suitable wealth transfer from older age-groups to age-group 1. (The working paper version (Kollmann, 2022) shows that a constant consumption share λ_1 is achieved when the transfer is set such that each newborn cohort's wealth represents a time-invariant share of aggregate wealth.) λ_1 is here treated as exogenous. The following analysis focuses on equilibria in which the (endogenous) consumption shares of age-groups $i > 1$ too are time-invariant, $\lambda_{i,t} = \lambda_i$.[2] Given λ_1, those steady state shares for $i > 1$ are uniquely determined and obey

$$\lambda_{i+1} = \lambda_i(1-\lambda_1)/(1-\lambda_N) \quad \text{for} \quad i = 1, \ldots, N-1. \tag{5}$$

2.2. Aggregate Euler equation

Only date t age-groups $i = 1, \ldots, N-1$ can hold assets between t and $t+1$. Let r_{t+1} be the rate of return of a traded asset, from t to $t+1$. The date t Euler equation of age-group $i = 1, \ldots, N-1$ for that asset is $E_t \rho_{t,t+1} \times (1 + r_{t+1}) = 1$, where $\rho_{t,t+1} = \beta c_{i,t}/c_{i+1,t+1}$ is the intertemporal marginal rate of substitution (IMRS). Full risk sharing implies that the IMRS is *equated* across those age-groups (see (2)). Thus,

$$\begin{aligned}\rho_{t,t+1} &= \beta \sum_{i=1}^{N-1} c_{i,t} \Big/ \sum_{i=2}^{N} c_{i,t+1} = \beta(C_t - c_{N,t})/(C_{t+1} - c_{1,t+1}) \\ &= \beta[(1-\lambda_N)/(1-\lambda_1)] \cdot C_t/C_{t+1}.\end{aligned}$$

Therefore,

$$\begin{aligned}&E_t \widetilde{\beta}(C_t/C_{t+1}) \times (1 + r_{t+1}) = 1, \\ &\text{with } \widetilde{\beta} \equiv \beta \times (1-\lambda_N)/(1-\lambda_1).\end{aligned} \tag{6}$$

$\widetilde{\beta}$ is increasing in λ_1. Intuitively, a higher age-group 1 consumption share implies lower growth of *individual* consumption over the life-cycle, which must be accompanied by a lower steady state interest rate, and a higher $\widetilde{\beta}$. $\lambda_1 = 1/N$ implies $\lambda_i = 1/N$ for $i > 1$, which entails $\widetilde{\beta} \equiv \beta$.[3] $\widetilde{\beta} > \beta$ holds, thus, when $\lambda_1 > 1/N$.

(6) shows that the OLG model implies a (quasi-) Euler equation, in terms of *aggregate* consumption, that has the same form as the Euler equation of an infinitely-lived representative agent (RA). However, an important difference, relative to RA models, is that the subjective discount factor in the aggregate OLG Euler equation $\widetilde{\beta}$ can exceed unity. Thus, the OLG economy can generate a low (even negative) steady state real interest rate, $r = (1-\widetilde{\beta})/\widetilde{\beta}$.[4]

In the OLG structure, each *individual* agent holds zero assets, at the end of her life, but the path of *aggregate* asset holdings is

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[1] For overviews of OLG models see, e.g., Guesnerie and Woodford (1992) and Niepelt (2019). In recent decades, advanced countries' short-term real interest rates have been very low by historical standards; analyses of low rates often use OLG models (Carvalho et al., 2016; Albonico et al., 2021).

[2] When consumption shares (for $i > 2$) *differ* from steady state shares, at some date, then equilibrium shares at later dates converge asymptotically to steady state consumption shares (given a constant λ_1).

[3] (5) implies $\lambda_{i+1} = \lambda_1 Q^i$, with $Q \equiv (1-\lambda_1)/(1-\lambda_N)$. (4) gives $\lambda_1 \sum_{i=1}^{N} Q^{i-1} = 1$. This pins down Q and λ_N as functions of λ_1. Q is decreasing in λ_1, so $\widetilde{\beta} \equiv \beta/Q$ is increasing in λ_1. $\lambda_1 = 1/N$ implies $Q = 1$ and $\lambda_i = 1/N$ for $i > 1$.

[4] $\widetilde{\beta} > 1$ can hold even when $\beta < 1$, provided $(1-\lambda_N)/(1-\lambda_1)$ is sufficiently large. Assume, e.g., $N = 320$ quarters (80 year life-span), and $\beta = 0.99$; then $\lambda_1 = 1.0373\%$ implies $\widetilde{\beta} = 1.005$ (steady state real interest rate: -2% per annum).

not constrained by a terminal condition—there is no aggregate transversality condition (TVC). By contrast, in an RA economy, optimizing individual behavior requires that a TVC for aggregate asset stocks holds.

3. Embedding the OLG structure in DSGE models

The above OLG structure can easily be built into a wide range of DSGE macro models. Assume that labor and the assets of different age-groups are homogeneous. Then the *static* equilibrium conditions, the budget constraints, and the laws of motion of individual asset holdings can be aggregated across age cohorts, which delivers equations in aggregate variables that are identical to corresponding equations in RA-DSGE models (see Kollmann, 2022 for further discussion). The upshot is that one can transform an RA-DSGE model into an OLG-DSGE model (with the OLG structure here) just by dropping the TVC. The format of the other aggregate equations remains unchanged, but the RA subjective discount factor $\beta < 1$ is replaced by the parameter $\widetilde{\beta}$ that may exceed unity (see (6)). As an illustration, I next discuss a New Keynesian (NK) economy that embeds the novel OLG structure.

4. An OLG-NK model

Consider the three-equation textbook NK model (e.g., Galí, 2015; Kollmann, 2021):

$$\pi_t = \kappa \cdot x_t + \widetilde{\beta} E_t \pi_{t+1}; \quad i_{t+1} = \gamma \pi_t; \quad E_t x_{t+1} = x_t + \{i_{t+1} - E_t \pi_{t+1} - r_{t+1}\}, \tag{7}$$

where π_t, x_t are inflation and the output gap at date t; i_{t+1}, r_{t+1} are the nominal interest rate, and the exogenous (stationary) natural real interest rate (i.e. the real interest rate in a flex-prices economy) between t and $t+1$. (All variables are expressed as deviations from steady state.) The three equations in (7) are a Phillips equation, a monetary policy (Taylor) rule and an Euler equation, respectively. $\kappa \geq 0$ is the slope of the Phillips curve. The Taylor principle is assumed to hold: $\gamma > 1$. When the OLG structure developed in this paper is assumed, the subjective discount factor in the Phillips equation is $\widetilde{\beta}$.[5] When $\widetilde{\beta} < 1$, the OLG-NK economy has a unique non-explosive equilibrium, for all $\kappa \geq 0, \gamma > 1$. When $\widetilde{\beta} \geq 1$, by contrast, multiple non-explosive equilibria may exist. E.g., one sees immediately that, for $\kappa = 0$, the processes $\pi_{t+1} = (1/\widetilde{\beta})\pi_t + \varepsilon_{t+1}, x_{t+1} = x_t + \{[\gamma - (1/\widetilde{\beta})]\pi_t - r_{t+1}\} + \eta_{t+1}$ satisfy (7), for *arbitrary* disturbances $\varepsilon_{t+1}, \eta_{t+1}$ with $E_t\varepsilon_{t+1} = E_t\eta_{t+1} = 0$; these inflation and output gap processes are non-explosive when $\widetilde{\beta} \geq 1$, which confirms the equilibrium indeterminacy. When $\kappa > 0$, equilibrium indeterminacy arises for values of $\widetilde{\beta}$ strictly larger than unity.[6] Even if the Taylor principle holds, the OLG-NK model may, thus, be indeterminate.

Data availability

No data was used for the research described in the article.

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[5] The Phillips equation follows from the first-order conditions of profit maximizing firms facing convex price adjustment costs. At t, firms optimally trade off price adjustment costs at t and $t+1$. Assume that firms evaluate this trade-off using the IMRS of households alive at t and $t+1$. Then $\widetilde{\beta}$ is the relevant subjective discount factor in the OLG Phillips equation.

[6] For empirically relevant values of κ and γ ($0 < \kappa \leq 0.5$; $1 < \gamma \leq 5$) the model is indeterminate when $\widetilde{\beta} \geq 1 + \gamma\kappa$.