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A DYNAMIC EQUILIBRIUM MODEL OF INTERNATIONAL PORTFOLIO HOLDINGS: COMMENT

BY ROBERT KOLLMANN[1]

1. INTRODUCTION

ONE OF THE MAIN PUZZLES in international finance is portfolio home bias: typical investors hold most of their wealth in domestic assets (despite the fact that international diversification permits a sizable reduction in risk). Serrat (2001) claims that the nontradable nature of many consumption goods explains home bias.

He presents a model of a world with two countries that receive tradable and nontradable endowments; financial transactions are restricted to trade in stocks that are claims to these endowments.[2] Serrat solves for a Pareto efficient equilibrium of that economy.

Serrat makes the following assertions: (i) The equilibrium portfolio is *unique* (p. 1472; Theorem 2). (ii) Claims to a country's traded good endowment are mainly held by local investors (for plausible correlations between endowments; see Table II). (iii) Claims to a country's nontraded good endowment are *only* held by local investors (Theorem 2).

This note shows that those assertions are incorrect: (i) Serrat's model fails to explain home bias. Due to the assumed preferences, dividends (evaluated at equilibrium goods prices) and equity prices are collinear in Serrat's economy. Thus, portfolios are indeterminate: the model does not pin down what fractions of the claims to a country's traded and nontraded good endowments are owned by local investors, what fraction of her wealth an investor allocates to domestic assets, or what fraction she allocates to claims to traded goods—these fractions can take any values. However, the model predicts that each investor's holdings of claims to traded goods are fully diversified internationally (each investor holds a share in the domestic traded goods sector that equals her share in the foreign traded goods sector); this prediction is counterfactual. (ii) The portfolio holdings described by Serrat (Theorem 2) do *not* implement the efficient allocation.

I next summarize the model features that are needed to establish these points.

2. ENDOWMENTS, PREFERENCES, PRICES

There are four perishable goods, $j = 1, \dots, 4$. Goods 1 and 2 [3 and 4] are tradable [nontradable].

[1] I thank a co-editor and two referees for advice, and Philippe Weil for helpful discussions.

[2] Serrat also assumes risk-free bonds; however, bonds are redundant in equilibrium.

Country 1 [2] receives endowments of goods 1 and 3 [2 and 4]. The good j endowment at date t is $\delta_{j,t}$, country i's ($i = 1, 2$) consumption of good j is $c^i_{j,t}$. The utility functions of countries 1 and 2 are $V^1_t = \frac{1}{q}(c^1_{3,t})^p[(c^1_{1,t})^q + (c^1_{2,t})^q]$ and $V^2_t = \frac{1}{q}(c^2_{4,t})^p[(c^2_{1,t})^q + (c^2_{2,t})^q]$, respectively, with $p + q < 1$, $pq > 0$.[3]

Serrat shows that a Pareto efficient equilibrium has the following properties:

(i) Country 1 consumes a fraction $\alpha_t = \{1 + [\Lambda(\delta_{4,t}/\delta_{3,t})^p]^{1/(1-q)}\}^{-1}$ of the world supply of tradables, where $\Lambda > 0$ is a time-invariant term that reflects countries' relative wealth: $c^1_{j,t} = \alpha_t \delta_{j,t}$, $c^2_{j,t} = (1 - \alpha_t)\delta_{j,t}$ for $j = 1, 2$. (Nontradables are consumed locally: $c^1_{3,t} = \delta_{3,t}$; $c^2_{4,t} = \delta_{4,t}$.)

(ii) $p_{2,t} = (\delta_{2,t}/\delta_{1,t})^{q-1}$, $p_{3,t} = (p/q)(1/\delta_{3,t})\alpha_t[\delta_{1,t} + (\delta_{1,t})^{1-q}(\delta_{2,t})^q]$, and $p_{4,t} = (p/q)(1/\delta_{4,t})(1 - \alpha_t)[\delta_{1,t} + (\delta_{1,t})^{1-q}(\delta_{2,t})^q]$, where $p_{j,t}$ is the price of good j (good 1 is the numéraire: $p_{1,t} \equiv 1$).

3. PORTFOLIOS

For simplicity, I first discuss time-invariant share holdings. Let S^i_j be a constant share in the good j endowment stream held by country i, with $S^1_j + S^2_j = 1$. With constant share holdings, a country's consumption spending equals her dividend income. Thus, constant share holdings implement the efficient allocation if

$$\sum_{j=1}^{4} p_{j,t} c^i_{j,t} = \sum_{j=1}^{4} S^i_j p_{j,t} \delta_{j,t}, \tag{1}$$

for $i = 1, 2$, and for all dates and states of nature, where $c^i_{j,t}$ and $p_{j,t}$ pertain to the efficient equilibrium. Plugging the above formulae for $c^1_{j,t}$, $p_{j,t}$ into (1), for $i = 1$, gives

$$\begin{aligned} 0 = &\left(S^1_1 + \frac{p}{q}S^1_4\right)\delta_{1,t} + \left(S^1_2 + \frac{p}{q}S^1_4\right)(\delta_{1,t})^{1-q}(\delta_{2,t})^q \\ &+ \left[\frac{p}{q}(S^1_3 - S^1_4) - 1 - \frac{p}{q}\right]\alpha_t[\delta_{1,t} + (\delta_{1,t})^{1-q}(\delta_{2,t})^q]. \end{aligned} \tag{2}$$

Equation (2) holds for random realizations of $\delta_{1,t}$, $\delta_{2,t}$, $\delta_{3,t}$, $\delta_{4,t}$ if and only if

$$S^1_2 = S^1_1, \quad S^1_3 = 1 + \frac{q}{p} - \frac{q}{p}S^1_1, \quad S^1_4 = -\frac{q}{p}S^1_1. \tag{3}$$

[3]Serrat's theoretical analysis allows for country-specific utility weights on tradables $p^1 \neq p^2$ (but he sets $p^1 = p^2$ in his numerical simulations); my key results go through when $p^1 \neq p^2$.

Equation (3) implies that (1) holds for country $i = 2$ as well, with shares $S^2_j = 1 - S^1_j$. Thus, *any* stock holdings $S^1_1, S^1_2, S^1_3, S^1_4$ consistent with (3) implement the efficient allocation.

Portfolio indeterminacy follows from the fact (apparently overlooked by Serrat) that dividends (in units of the numéraire) are collinear, because preferences are Cobb–Douglas in terms of nontradables and a composite of tradables: $p_{3,t}\delta_{3,t} + p_{4,t}\delta_{4,t} = \frac{p}{q}[\delta_{1,t} + p_{2,t}\delta_{2,t}]$.

Portfolio indeterminacy implies that Serrat's (p. 1478) measures of home bias (HBT_t and HB_t: are proportions of domestic traded good firm and of total domestic firms held by domestic investors; $HBT^1_t \equiv S^1_{1,t}$ and $HB^1_t \equiv (S^1_{1,t}P_{1,t} + S^1_{3,t}P_{3,t})/(P_{1,t} + P_{3,t})$, where $P_{j,t}$ is the price of stock j) can take any value between $-\infty$ and $+\infty$. However, each investor's portfolio of "traded good equities" is fully diversified internationally: $S^i_2 = S^i_1$. This prediction is counterfactual.[4]

4. ISOLATING THE ERROR IN SERRAT'S ANALYSIS

Under Serrat's assumption that endowments follow a diffusion process, equilibrium portfolios have to satisfy the equation, for $i = 1, 2$ (see p. 1484),

$$(4) \qquad \Lambda_t \pi^{i\prime}_t = \widehat{\Phi}^{i\prime}_t \quad \text{with} \quad \pi^i_t \equiv (\pi^i_{1,t}, \pi^i_{2,t}, \pi^i_{3,t}, \pi^i_{4,t}), \quad \pi^i_{j,t} \equiv S^i_{j,t}P_{j,t},$$

where $\pi^i_{j,t}$ is the value of the stock j share held by country i at t ($S^i_{j,t}$) and Λ_t [$\widehat{\Phi}^i_t$] is a 4×4 [1×4] matrix. (The diffusion matrices of the vector of endowments, of the vector of stock returns, and of the present value of country i's efficient consumption spending are σ, $\Lambda'_t\sigma$, and $\widehat{\Phi}^i_t\sigma$, respectively. Equation (4) ensures that i's portfolio finances i's efficient consumption process.)

Serrat fails to correctly solve (4) for π^i_t (see Kollmann (2005) for more detailed discussions):

(i) Matrix Λ_t is singular (because stock prices are collinear, due to collinearity of dividends (in units of numéraire)). Thus, the solution of (4) is *not* unique.

(ii) Equation (4) holds if and only if

$$(5) \qquad S^1_{2,t} = S^1_{1,t}, \quad S^1_{3,t} = 1 + \frac{q}{p} - \frac{q}{p}S^1_{1,t}, \quad S^1_{4,t} = -\frac{q}{p}S^1_{1,t}, \quad \text{and}$$
$$S^2_{j,t} = 1 - S^1_{j,t} \quad \text{for } j = 1, \dots, 4.$$

Any sequence $\{S^1_{1,t}, S^1_{2,t}, S^1_{3,t}, S^1_{4,t}\}$ consistent with (5) supports the efficient equilibrium. (Note that the time-invariant stock holdings (3) satisfy (5).)

[4]Empirically, there is home bias for manufacturing equity (manufactured goods: traded); see, e.g., Kang and Stulz (1995), who document that Japanese investors held more than 86% of Japanese manufacturing equity during 1975–1991. See also Baxter, Jermann, and King (1995).

PROOF OF (ii): If the stock market clears and (4) holds for $i = 1$, then (4) holds for $i = 2$ too. Here, I solve (4) for $i = 1$. The fourth row of Λ_t is proportional to its third row and $\widehat{\Phi}_t^{1\prime}$ equals the third column of Λ_t times $(1 + \frac{q}{p})P_{3,t}$. Thus, (4) holds if and only if

$$\widehat{\Lambda}_t\left(P_{1,t}S^1_{1,t}, P_{2,t}S^1_{2,t}, P_{3,t}\left(S^1_{3,t} - \left(1 + \frac{q}{p}\right)\right), P_{4,t}S^1_{4,t}\right)' = 0, \tag{6}$$

where $\widehat{\Lambda}_t$ is the matrix that consists of the first three rows of Λ_t. Premultiplying (6) by the nonsingular matrix

$$\begin{bmatrix} 1 & 1 - 1/q & 0 \\ 0 & 1/q & 0 \\ \alpha_t - u^1_t & K_{1,t} & 1/p \end{bmatrix}$$

gives

$$\begin{bmatrix} 1 & 0 & 1 - b^1_t & 1 - b^2_t \\ 0 & 1 & b^1_t & b^2_t \\ 0 & 0 & K_{2,t} & K_{3,t} \end{bmatrix} \times \left(P_{1,t}S^1_{1,t}, P_{2,t}S^1_{2,t}, P_{3,t}\left(S^1_{3,t} - \left(1 + \frac{q}{p}\right)\right), P_{4,t}S^1_{4,t}\right)' = 0_3, \tag{7}$$

where $K_{1,t} \equiv \alpha_t - u^1_t + [u^1_t - u^2_t]\frac{1}{q}$; the variables u^1_t, u^2_t, b^1_t, b^2_t (defined by Serrat, p. 1483) satisfy $u^1_t P_{1,t} + u^2_t P_{2,t} = \frac{q}{p}P_{3,t}$ and $b^1_t P_{3,t} + b^2_t P_{4,t} = \frac{p}{q}P_{2,t}$; and $K_{2,t}$ and $K_{3,t}$ are terms with $K_{2,t}P_{3,t} = -K_{3,t}P_{4,t} \neq 0$. Therefore, (7) holds if and only if $S^1_{3,t} - (1 + \frac{q}{p}) = S^1_{4,t}$, $S^1_{2,t} = -\frac{p}{q}S^1_{4,t}$, and $S^1_{1,t} = S^1_{2,t}$, which is equivalent to (5). *Q.E.D.*

(iii) The portfolio holdings described by Serrat ($S^1_{3,t} = 1$, $S^1_{4,t} = 0$; see (17) in his Theorem 2) are inconsistent with (5). Thus, those holdings do *not* support the efficient consumption processes. (Substitution of Serrat's (17) into (4) confirms that his portfolio holdings do not solve (4).)

Dept. of Economics, University of Paris XII, F-94010 Créteil Cedex, France; and CEPR; robert_kollmann@yahoo.com; http://www.robertkollmann.com.

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