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Solving the multi-country Real Business Cycle model using a perturbation method

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ABSTRACT

This paper solves the multi-country RBC model described in den Haan et al. (this issue) and Juillard and Villemot (this issue), using a perturbation method. We explain how to apply first- and second-order versions of the gensys2.m algorithm to this model. The perturbation method is computationally cheap and can easily be applied to large models with possibly hundreds of state variables.

1. Introduction

This paper explains how we solve the multi-country RBC model described in den Haan et al. (this issue) and Juillard and Villemot (this issue), using first- and second-order perturbation methods. These methods are represented by PER1 (first-order perturbation) and PER2 (second-order perturbation) in the comparison paper of Kollmann et al. (this issue).

Perturbation methods solve the coefficients of Taylor expansions of the true model solution around a deterministic steady state. Compared to projection-based non-linear techniques (e.g., Judd, 1998), perturbation methods have two key advantages: their high computational speed and the ease with which they can be applied to models with a large number of state variables. This explains why many dynamic stochastic general equilibrium (DSGE) models have been solved using perturbation methods. First-order (i.e., linearization) techniques have been most widely used in the macroeconomics literature. Recently, however, a rapidly growing number of studies have applied second-order perturbation methods, thanks to several publicly available solution algorithms.[1]

For applying perturbation methods, we use the MATLAB algorithm gensys2.m described in Sims (2002), Kollmann (2003b) and Kim et al. (2008).[2] Other solution algorithms for second-order perturbation include Jin and Judd (2004),

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[1] For example, the second-order perturbation method is widely used for welfare analyses of monetary and fiscal policy rules.

[2] The authors of the present paper participated in the development of the code, which can be found on Chris Sims' webpage: http://sims.princeton.edu/yftp/gensys2. For applications of this specific second-order perturbation algorithm, see for example Bergin and Tchakarov (2007); Kim and Kim (2003a, 2003b); Kim (2004); Kim et al. (2010); Kollmann (2002, 2003a, 2004, 2008); Marzo (2004); Shin (2004); Straub and Tchakarov (2005) and Teo (2003).

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Schmitt-Grohé and Uribe (2004); Anderson et al. (2006); Lombardo and Sutherland (2007) and Dynare by Juillard et al. (2010).[3]

Although perturbation methods are computationally cheap, accuracy may be lower, especially when the model economy moves significantly away from the steady state. Between the two versions considered in this paper, the second-order perturbation solution can be noticeably more accurate than the first-order perturbation solution (see Kollmann et al., this issue, for detailed results).

2. Solution algorithm

The gensys2.m code can be applied to the models of the following form:

$$E_t \Psi(\omega_{t+1},\omega_t,\varepsilon_{t+1}) = 0 \tag{1}$$

where ω_t is a $q\times 1$ vector of endogenous and exogenous variables known at date t, while ε_{t+1} is a vector of exogenous disturbances with $E_t\varepsilon_{t+1}=0$ and $E_t\varepsilon_{t+1}\varepsilon'_{t+1}=\Omega$.

We assume that the model has a unique deterministic steady state $\overline{\omega}$ (satisfying $\Psi(\omega,\overline{\omega},0)=0$), and also that the solution of Eq. (1) is unique and of the form

$$y_{t+1} = F(y_t,\varepsilon_{t+1}) \tag{2}$$

$$x_{t+1} = M(y_{t+1}) \tag{3}$$

where y_t ($q_s\times 1$ vector) and x_t ($(q-q_s)\times 1$ vector) are linear combinations of the original variables ω_t: $[y_t;x_t]=Z\omega_t$, for some square, non-singular matrix Z. Note that y_t and x_t can be interpreted as internally generated state and control variables, respectively.[4] Users of the gensys2.m code do not need to specify which variables are state variables and which ones are controls.[5]

The solution can also be expressed in terms of original variables ω_t as follows:

$$\omega_{t+1} = \Phi(\omega_t,\varepsilon_{t+1}) \equiv Z^{-1}\begin{bmatrix} F(Z_1\omega_t,\varepsilon_{t+1}) \\ M(F(Z_1\omega_t,\varepsilon_{t+1})) \end{bmatrix} \tag{4}$$

where Z_1 is a matrix consisting of the first q_s rows of the matrix Z ($y_t=Z_1\,\omega_t$).

The gensys2.m code constructs second degree polynomials, which approximate Eqs. (2) and (3), in the neighborhood of the deterministic steady state. The coefficients of these polynomials are functions of Ω and of the first and second derivatives of $\Psi(\omega_{t+1},\omega_t,\varepsilon_{t+1})$ evaluated at the steady state. Let $y_{t+1}=\hat{F}(y_t,\varepsilon_{t+1})$, $x_{t+1}=\hat{M}(y_{t+1})$ denote the second-order polynomials that approximate Eqs. (2) and (3). Then

$$\omega_{t+1} \simeq \hat{\Phi}(\omega_t,\varepsilon_{t+1}) \equiv Z^{-1}\begin{bmatrix} \hat{F}(Z_1\omega_t,\varepsilon_{t+1}) \\ \hat{M}(\hat{F}(Z_1\omega_t,\varepsilon_{t+1})) \end{bmatrix} \tag{5}$$

3. Application to the model in the comparison project

We use log variables for perturbing the model in the comparison project; that is, the approximation is taken in terms of the following variables:

$$\omega_t = (\ln(\lambda_t),\ \ln(c_t^1),\ \dots,\ \ln(c_t^N);\ \ln(l_t^1),\ \dots,\ \ln(l_t^N);\ \ln(i_t^1),\ \dots,\ \ln(i_t^N);\ \ln(k_{t+1}^1),\ \dots,\ \ln(k_{t+1}^N);\ \ln(a_t^1),\ \dots,\ \ln(a_t^N))$$

where λ_t is the Lagrange multiplier of the world resource constraint, while c_t^i, l_t^i, $i_t^i$and $a_t^i$are consumption, hours worked, investment and total factor productivity in country i, respectively. N is the number of countries. The vector of the countries' log TFP's follows an AR(1) process with a vector of innovations ε_{t+1}.

We use a two-point finite difference procedure (Fackler and Miranda, 2002, p. 98 and p. 102) to compute the first and second derivatives of $\Psi(\omega_{t+1},\omega_t,\varepsilon_{t+1})$.[6]

The comparison paper requires the computation of a policy function that expresses the date t+1 endogenous non-predetermined variables as a function of capital stocks at the beginning of t+1, and of productivity at t+1 (in N countries). Let

$$z_t \equiv (\ln(\lambda_t),\ \ln(c_t^1),\ \dots,\ \ln(c_t^N);\ \ln(l_t^1),\ \dots,\ \ln(l_t^N);\dots \ln(i_t^1),\ \dots,\ \ln(i_t^N))$$

$$K_{t+1} \equiv (\ln(k_{t+1}^1),\ \dots,\ \ln(k_{t+1}^N))$$

[3] Some algorithms are available that can perform third- (or higher-) order perturbation, e.g. Dynare++ and Jin and Judd (2004). However, these higher-order algorithms have not yet been applied to large DSGE models such as those used in central banks or the multi-country model in this paper. Evaluating the accuracy of such higher-order algorithms for large models would be an interesting topic for future research.

[4] The notation here follows Kim et al. (2008). By contrast, Schmitt-Grohé and Uribe (2004) denoted state variables by x and control variables by y.

[5] This state-free approach is also adopted by Anderson et al. (2006) and Dynare, while other algorithms such as those by Schmitt-Grohé and Uribe (2004) and Lombardo and Sutherland (2007) required users to specify the partition between the state and control variables as an input to the algorithm.

[6] The derivatives could also be computed using the MATLAB symbolic toolbox. The two-point finite difference procedure is computationally faster; the differences between the two methods turn out to be numerically insignificant in this model.

and

$$A_t \equiv (\ln(a_t^1), \ldots, \ln(a_t^N))$$

As $\omega_t=(z_t, K_{t+1}, A_t)$, solution (5) can be transformed into the following policy function:

$$\omega_{t+1} \simeq \hat{\Phi}((z_t, K_{t+1}, A_t), \varepsilon_{t+1}) = \hat{H}(K_{t+1}, A_{t+1}) \tag{6}$$

where we use the fact that the influence of A_t and ε_{t+1} on ω_{t+1} can be subsumed by A_{t+1} (due to the AR(1) structure of the vector of log TFP's, A_t).

All results generated by PER1 and PER2 in the comparison paper are based on dynamic simulations of Eq. (6). (The first-order solution PER1 is generated by setting the second-order coefficients of the policy function—including the constant term that is affected by the amount of uncertainty—to zero.) In other words, the simulations for the comparison paper do not use the pruning technique of Kim et al. (2008).[7] For the model variants in the comparison paper, the simulated series without pruning (10,000 periods) do not differ noticeably from pruned series.[8]

While our results reported in the comparison paper use logged variables as perturbation variables, as a sensitivity analysis we also solved the model using the levels as perturbation variables.[9] The motivation for this alternative approach is that the law of motion for capital is linear in the levels of capital and investment; thus this law of motion is exactly (i.e., without any approximation error) captured by employing the levels as a perturbation variable. Inspection of the errors in the individual model equations shows that the errors in the Euler equation and the world resource constraint are roughly unaffected when a level approximation is used, but that the errors in the risk sharing and labor supply equations increase (compared to the log approximation). Maximum errors across all model equations typically change little. For example, in values of state variables visited along 10,000-period stochastic simulation runs, we find that, across all equations of all 'asymmetric' model specifications (in which preferences/technology parameters differ across countries), the maximum absolute error is 6.30% under a first-order log approximation, compared to 4.57% under a first-order level approximation.

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[7] A second-order approximation of a difference equation such as Eq. (4) has extraneous steady states (that are not present in the original model), and some of these steady states mark transitions to unstable behavior. Large shocks can thus move the model into an unstable region. The pruning procedure overcomes this problem. It is motivated by the observations that in repeated applications of the second-order perturbation solution (6), third or higher-order terms of state variables appear. For example, when K_{t+2} is quadratic in K_{t+1}, then K_{t+2} is quartic in K_t. The pruning procedure removes these higher-order terms by computing the second-order terms using the squares of the linearized solution. Since the first-order expansion is stable, the pruned version of a second-order approximation achieves stability.

[8] Explosive paths typically emerge in non-pruned 10,000-period simulations only when the standard deviation of shocks is set at an order-of-magnitude larger than specified in the comparison paper.

[9] Specifically, we approximate the Lagrange multiplier of the world resource constraint, consumption, hours worked, investment and capital in levels. However, we continue to approximate TFP in logs (as the law of motion of TFP is linear in logs).

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