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NOT-FOR-PUBLICATION APPENDIX

The Post-Crisis Slump in the Euro Area and the US: Evidence from an Estimated Three-Region DSGE Model

Robert Kollmann, Beatrice Pataracchia, Rafal Raciborski, Marco Ratto, Werner Roeger, Lukas Vogel

A. Detailed model description and parameter estimates

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Appendix A: Detailed model description and parameter estimates

We consider a three-country world consisting of the Euro Area (EA), the United States (US), and the rest of the world (RoW). The EA and US blocks of the model are rather detailed, while the RoW block is more stylized.[1] The EA and US blocks assume two (representative) households, a number of layers of firms and a government. EA and US households provide labor services to firms. One of the two households (savers, or 'Ricardians') in each country has access to financial markets, and she owns her country's firms. The other (liquidity-constrained, or 'non-Ricardian') household has no access to financial markets, does not own financial or physical capital, and in each period only consumes the disposable wage and transfer income. The preferences of both types of household exhibit habit formation in both consumption and leisure, a feature which allows for better capturing persistence of the data.

There is a monopolistically-competitive sector producing differentiated goods in the EA and the US, using domestic labor and capital and being able to. The firms in the sector maximize the present value of dividends at a discount factor that is strictly larger than the risk-free rate and varies over time. This is a short-cut for capturing financial frictions facing firms; it can, e.g., be interpreted as a 'principal agent friction' between the owner and the management of the firm (e.g., Hall (2011)). Optimization is subject to investment and labor adjustment costs and a varying capacity utilization rate, which lets the model better capture the dynamics of the current account and other macro variables.

Total output in the EA and the US is produced by combining the domestic differentiated goods bundle with energy input. EA and US wages are set by monopolistic trade unions. Nominal differentiated goods prices are sticky as are the wages paid to the workers. Fiscal authorities in the EA and the US impose distortive taxes and issue debt.

The RoW block is simplified compared to the US and EA blocks. Specifically, the RoW consists of a budget constraint for the representative household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses labor as the sole factor input, and a New Keynesian Phillips curve. The RoW block abstracts from capital accumulation.

The behavioral relationships and technology are subject to autocorrelated shocks denoted by ε_t^x , where x stands for the type of shock. ε_t^x will generally follow an AR(1) process with autocorrelation coefficient $\rho^x < 1$ and innovation u_t^x:

[1]The EA and US blocks build on, but are considerably different than the QUEST model of the EU economy (Ratto et al., 2009).

$$(\varepsilon_t^x) = \rho^x(\varepsilon_t^x) + u_t^x$$

There is also a separate category of shocks, denoted A_t^x, whose logs are integrated of order 1.[2] With the exception of the TFP shocks, these shocks are modelled as ARIMA(1,1,0) shocks.[3] We next present a detailed description of EA and US blocks,[4] followed by an overview of the RoW model block. Throughout the derivation the following indexing convention will be preserved. Indices *i* and *j* index firms and households, respectively. These indices will usually be dropped when the equilibrium conditions are derived due to the representative household/firm assumption. Index *l* indicates sovereign states or economic regions. Finally, index *k* will always indicate the 'domestic' economy. This index will be generally dropped for parameters (even if they are country-specific), but will be usually preserved for variables.

A.1. EA and US households

The household sector consists of a continuum of households $j \in [0;1]$. There are two types of households, savers ("Ricardians", superscript *s*) who own firms and hold government and foreign bonds and liquidity-constrained households (subscript *c*) whose only income is labor income and who do not save. The share of savers in the population is ω^s.

Both households enjoy utility from consumption C_{jkt}^r and incur disutility from labor N_{jkt}^r ($r = s, c$). On top of this, Ricardian's utility depends also on the financial assets held.

Date *t* expected life-time utility of household *r*, is defined as:

$$U_{jkt}^r = \sum_{s=t}^{\infty} \varepsilon_{kt}^c \beta^{s-t} u_{jkt}^r(\cdot)$$

where β is the (non-stochastic) discount factor (common for both types of households) and ε_{kt}^c is the saving shock.

A.1.1. Ricardian household

The Ricardian households work, consume, own firms and receive nominal transfers T_{jkt}^s from the government. Ricardians have full access to financial markets and are the only households who own financial assets $\frac{A_{jkt}}{P_{kt}^{c,vat}}$ where $P_{kt}^{c,vat}$ is consumption price, including VAT.[5] Financial

[2] These, in particular, include the TFP shock and the final demand productivity shocks.
[3] TFP is driven by 3 shocks, see below.
[4] The EA and US blocks have the same structure. The parameter values for the equations are country-specific as determined in the estimation.
[5] Note that $P_{kt}^{c,vat}$ is related to P_{kt}^C, the private consumption deflator in terms of input factors, by the formula: $P_{kt}^{c,vat} = (1+\tau_k^C)P_{kt}^C$ where τ^c is the tax on consumption.

wealth of household j consists of bonds $\frac{B_{jkt}}{P_{kt}^{c,vat}}$ and shares $\frac{P_{kt}^{S}S_{jkt}}{P_{kt}^{c,vat}}$, where P_{kt}^{S} is the nominal price of shares in t and S_{jkt} the number of shares held by the household:

$$\frac{A_{jkt}}{P_{kt}^{c,vat}} = \frac{B_{jkt}}{P_{kt}^{c,vat}} + \frac{P_{kt}^{S}S_{jkt}}{P_{kt}^{c,vat}}$$

It is assumed that households invest only in domestic shares. Bonds consist of government domestic $\frac{B_{jkkt}^{g}}{P_{kt}^{c,vat}}$ and foreign bonds $\frac{e_{lkt}\ B_{jlkt}^{g}}{P_{kt}^{c,vat}}$ and private risk-free bonds $\frac{B_{jkt}^{rf}}{P_{kt}^{c,vat}}$ (in zero supply):

$$\frac{B_{jkt}}{P_{kt}^{c,vat}} = \frac{B_{jkt}^{rf}}{P_{kt}^{c,vat}} + \sum\nolimits_{l} e_{lkt} B_{jlkt}^{g}$$

with e_{lkt} the bilateral exchange rate and $e_{kkt} \equiv 1$.[6] The budget constraint of a saver household j is:

$$(1-\tau^{N})W_{kt}N_{jkt}^{s} + \sum\nolimits_{l}\left(1+i_{lt-1}^{g}\right)e_{lkt}B_{jlkt-1}^{g} + \left(1+i_{t-1}^{rf}\right)B_{jkt-1}^{rf} + (P_{kt}^{S} + P_{kt}^{Y}d_{kt})S_{jkt-1} + div_{kt} + T_{jkt}^{s} - tax^{s}{}_{jkt} = P_{kt}^{c,vat}C_{jkt}^{s} + A_{jkt}$$

where W_{kt} is the nominal wage rate, P_{kt}^{Y}, is GDP price deflator, i_{lt-1}^{g} are interest rates on government bonds of region l, i_{t-1}^{rf} is interest rate on risk-free bond, T_{jkt}^{s} are government transfers to savers and $tax^{s}{}_{jkt}$ are lump-sum taxes paid by savers. Note that savers own all the firms in the economy. div_{kt} represent the profits of all firms other than differentiated goods producers (the latter producers transfer profits to savers by paying dividends d_{kt}).

We define the gross nominal return on domestic shares as:

$$1 + i_{kt}^{s} = \frac{P_{kt}^{S} + P_{kt}^{Y}d_{kt}}{P_{kt-1}^{S}}$$

The instantaneous utility functions of savers, $u^{s}(\cdot)$, is defined as:

$$u^{s}\left(C_{jkt}^{s}, N_{jkt}^{s}, \frac{U_{jkt-1}^{A}}{P_{kt}^{c,vat}}\right) = \frac{1}{1-\theta}\left(C_{jkt}^{s} - hC_{kt-1}^{s}\right)^{1-\theta} - \frac{\omega^{N}\varepsilon_{kt}^{U}}{1+\theta^{N}}(C_{kt}^{s})^{1-\theta}\left(N_{jkt}^{s} - h_{N}N_{kt-1}^{s}\right)^{1+\theta^{N}} - \ (C_{kt}^{s} - hC_{kt-1}^{s})^{-\theta}\ \frac{U_{jkt-1}^{A}}{P_{kt}^{c,vat}}$$

[6] For simplicity, at this moment the model assumes only one type of foreign bonds, B_{RoWkt}^{g}, issued by RoW and denominated in RoW currency.

where $C_{kt}^{s}=\int C_{jkt}^{s},\ C_{kt}=\omega^{s}C_{kt}^{s}+(1-\omega^{s})C_{kt}^{c}\ ;\ h,h_{N}\in(0;1)$ measure the strength of the external habits in consumption and labor and $\varepsilon_{kt}^{U}$is the labor supply (or wage mark-up) shock . The disutility of holding financial assets, U_{jkt-1}^{A}, is defined as:

$$U_{jkt-1}^{A}=\sum_{l}\left(\left(\alpha_{lk}^{bB0}+\varepsilon_{lkt-1}^{B}\right)e_{lkt-1}B^{g}{}_{jlkt-1}\right)+$$

$$\left(\left(\alpha_{k}^{sS0}+\varepsilon_{kt-1}^{S}\right)P_{st-1}^{s}\,S_{jkt-1}\right)$$

The Ricardian household problem leads to the following first order conditions (FOC).[7]

The FOC w.r.t. savers' consumption produces:

$$\varepsilon_{kt}^{C}(C_{kt}^{s}-hC_{kt-1}^{s})^{-\theta}=\lambda_{kt}^{s}$$

where λ_{kt}^{s} is the Lagrange multiplier on the budget constraint.

FOC w.r.t. domestic risk-free bond:

$$\beta E_{t}\left[\frac{\lambda_{kt+1}^{s}}{\lambda_{kt}^{s}}\frac{1+i_{kt}^{rf}}{1+\pi_{kt+1}^{C,vat}}\right]=1$$

FOC w.r.t. domestic government bonds:

$$\beta E_{t}\left[\frac{\lambda_{kt+1}^{s}}{\lambda_{kt}^{s}}\frac{1+i_{kt}^{g}-\varepsilon_{kt}^{B}-\alpha_{kk}^{b0}}{1+\pi_{kt+1}^{C,vat}}\right]=1$$

with $\pi_{kt}^{C,vat}$ the consumption deflator inflation rate and ε_{kt}^{B} the risk-premium on government bonds.

FOC w.r.t. RoW government bonds:

$$\beta E_{t}\left[\frac{\lambda_{kt+1}^{s}}{\lambda_{kt}^{s}}\frac{\left(1+i_{RoWkt}^{g}\right)\frac{e_{RoWkt+1}}{e_{RoWkt}}-\varepsilon_{RoWkt}^{B}-\left(\alpha_{RoWk}^{b0}+\alpha_{RoWk}^{b1}\frac{e_{RoWkt}B_{RoWkt}^{g}}{P_{k}^{Y}Y_{k}}\right)}{1+\pi_{kt+1}^{C,vat}}\right]=1$$

where ε_{RoWkt}^{B} the risk-premium on RoW bonds.

FOC w.r.t. domestic stocks:

$$\beta E_{t}\left[\frac{\lambda_{kt+1}^{s}}{\lambda_{kt}^{s}}\frac{(1+i_{kt+1}^{s})-\varepsilon_{kt}^{S}-\alpha_{kk}^{s0}}{1+\pi_{t+1}^{C,vat}}\right]=1$$

where ε_{kt}^{S} the risk-premium on stocks. The above optimality conditions are similar to a textbook Euler equation, but incorporate asset-specific risk premia, which depend on an

[7] See subsection A.1.3 for the labor supply condition.

exogenous shock ε_{kt}^{A} as well as the size of the asset holdings as a share of GDP, see Vitek (2013, 2014) for a similar formulation. Taking into account the Euler equation for the risk-free bond and approximating, they simplify to the familiar expressions:

$$i_{kt}^{g} = i_{kt}^{rf} + rprem_{kt}^{g}$$

$$E_t\left[\frac{e_{RoWkt+1}}{e_{RoWkt}}\right] i_{RoWkt}^{g} = i_{kt}^{rf} + rprem_{RoWkt}^{g}$$

$$i_{kt}^{s} = i_{kt}^{rf} + rprem_{kt}^{s}$$

In the equations above, $rprem_{kt}^{g}$ is the risk premium on domestic government bonds. Similarly, $rprem_{RoWkt}^{g}$ is the risk premium on domestic government bonds sold abroad (to RoW). This feature of the model, hence, helps capture international spillovers that occur via the financial market channel, see Vitek (2013, 2014). Finally, $rprem_{kt}^{s}$ is a crucial risk premium on domestic shares. It is introduced to capture in a stylized manner financial frictions that are commonly believed to have contributed to the first phase of the financial crisis and may have contributed to its second phase, see also subsection A.2.2, below.[8]

A.1.2. liquidity-constrained household

The liquidity-constrained household consumes her disposable after-tax wage and transfer income in each period of time ('hand-to-mouth'). The period t budget constraint of the liquidity-constrained household is:

$$(1+\tau_k^C)P_{kt}^C C_{jkt}^c = (1-\tau_k{}^N)W_{kt}N_{kt}^c + T_{kt}^c - tax^c{}_{jkt}.$$

The instantaneous utility functions for liquidity-constrained households. $u^c(\cdot)$, is defined as:

$$u^c\left(C_{jkt}^c, N_{jkt}^c\right) = \frac{1}{1-\theta}\left(C_{jkt}^c - hC_{kt-1}^c\right)^{1-\theta} - \left(C_{kt}^c\right)^{1-\theta}\frac{\omega^N exp(u_{kt}^U)}{1+\theta^N}\left(N_{jkt}^c - h_N N_{kt-1}^c\right)^{1+\theta^N}$$

with $C_{kt}^c = \int C_{jkt}^c$.

A.1.3. Labor supply

Trade unions are maximizing a joint utility function for each type of labor. It is assumed that types of labor are distributed equally over Ricardian and liquidity-constrained households with their respective population weights. The wage rule is obtained by equating a weighted average of the marginal utility of leisure to a weighted average of the marginal utility of consumption times the real wage adjusted for a wage mark-up. Nominal rigidity in wage setting is introduced in the form of adjustment costs for changing wages. The wage adjustment costs are borne by the household. Real wage rigidity is also allowed, given the following optimality condition:

[8] Observationally, this approach is equivalent to exogenous risk premia as well as risk premia derived in the spirit of Bernanke, Gertler & Gilchrist.

$$\left((1+\mu_t^W)\frac{\omega^S V_{1-l,jkt}^S+(1-\omega^S)V_{1-l,jkt}^C}{\omega^S U_{c,jkt}^S+(1-\omega^S)U_{c,jkt}^C}(1+\tau_k^C)p_{kt}^C\right)^{1-\gamma^{wr}}\left((1-\tau_k{}^N)\frac{W_{kt-1}}{P_{kt-1}^Y}\right)^{\gamma^{wr}}=(1-\tau_k{}^N)\frac{W_{kt}}{P_{kt}^Y}+\gamma^w(\pi_t^W-(1-sf^w)\pi_{t-1}^W)(1+\pi_t^W)\text{-}\gamma^w\frac{L_{t+1}}{L_t}\frac{1+\pi_{t+1}^y}{1+i_{t+1}^{sd}}(\pi_{t+1}^W-(1-sf^w)\pi_t^W)(1+\pi_{t+1}^W)$$

where μ_t^W is the wage mark-up, γ^{wr} is the degree of real wage rigidity, γ^w is the degree of nominal wage rigidity and sf^w is the degree of forward-lookingness in the labor supply equation. $V_{N,jkt}^x$, for x=s,c, is the marginal disutility of labor, defined as:

$$V_{N,jkt}^x=\omega^N exp(u_{kt}^U)C_{kt}{}^{1-\theta}\left(N_{jkt}^x-h_N N_{kt-1}^x\right)^{\theta^N}$$

A.2. EA and US production sector

A.2.1. Total output demand

Total output O_{kt} is produced by perfectly competitive firms by combining value added, Y_{kt}, with energy input, Oil_{kt}, using the following CES production function:

$$O_{kt}=\left[\left(1-s^{Oil}\right)^{\frac{1}{\sigma^o}}(Y_{kt})^{\frac{\sigma^o-1}{\sigma^o}}+\left(s^{Oil}\right)^{\frac{1}{\sigma^o}}(OIL_{kt})^{\frac{\sigma^o-1}{\sigma^o}}\right]^{\frac{\sigma^o}{\sigma^o-1}}$$

where s^{Oil} is the energy input share in total output and elasticity σ^o is inversely related to the steady state output price gross mark-up. It follows that the demand for Y_{kt} and OIL_{kt} by total output producers is, respectively:

$$Y_{kt}=\left(1-s^{Oil}\right)\left(\frac{P_{kt}^Y}{P_{kt}^O}\right)^{-\sigma^o}O_{kt}$$

$$OIL_{kt}=s^{Oil}\left(\frac{P_{kt}^{Oil}}{P_{kt}^O}\right)^{-\sigma^o}O_{kt}$$

where P_{kt}^Y and $P_{kt}^{Oil}$are price deflators associated with Y_{kt} and Oil_{kt}, respectively, and the total output deflator P_{kt}^O is such that:

$$P_{kt}^O=\left[\left(1-s^{Oil}\right)(P_{kt}^Y)^{1-\sigma^o}+s^{Oil}\left(P_{kt}^{Oil}\right)^{1-\sigma^o}\right]^{\frac{1}{1-\sigma^o}}$$

A.2.2. Differentiated goods supply

Each firm $i\in[0;1]$ produces a variety of the domestic good which is an imperfect substitute for varieties produced by other firms. Because of imperfect substitutability, firms are monopolistically competitive in the goods market and face a downward-sloping demand function for goods. Domestic final good producers then combine the different varieties into a homogenous good and sell them to domestic final demand goods producers and exporters.

Differentiated goods are produced using total capital K_{ikt-1}^{tot} and labour N_{ikt} which are combined in a Cobb-Douglas production function:

$$Y_{ikt} = (A_{kt}^{Y} N_{ikt})^{\alpha} (cu_{ikt} K_{ikt-1}^{tot})^{1-\alpha}$$

where A_{kt}^{Y} is labour-augmenting productivity shock common to all firms in the differentiated goods sector and cu_{ikt} is firm-specific level of capital utilization. Total Factor Productivity, TFP_{kt}, can therefore be defined as:

$$TFP_{kt} = (A_{kt}^{Y})^{\alpha}.$$

We allow for three types of shocks related to the technology: a temporary shock ε_{kt}^{AY} which accounts for temporary deviations of A_{kt}^{Y} from its trend, $\bar{A}_{kt}^{Y}$, and two shocks related to the trend components itself:

$$\log(A_{kt}^{Y}) - log(\bar{A}_{kt}^{Y}) = \varepsilon_{kt}^{AY}$$

$$\log(\bar{A}_{kt}^{Y}) - log(\bar{A}_{kt-1}^{Y}) = g_{kt}^{\overline{AY}} + \varepsilon_{kt}^{L\overline{AY}}$$

$$g_{kt}^{\overline{AY}} = \rho^{\overline{AY}} g_{kt-1}^{\overline{AY}} + \varepsilon_{kt}^{G\overline{AY}} + \left(1 - \rho^{\overline{AY}}\right) g^{\overline{AY}}$$

with $g^{\overline{AY}}$ being the long-run technology growth.

Total capital is a sum of private installed capital, K_{ikt}, and public capital, K_{ikt}^{g}:

$$K_{ikt}^{tot} = K_{ikt} + K_{ikt}^{g}$$

The producers maximize the value of the firm, V_{kt}, equal to a discounted stream of future dividends, $V_{kt} = d_{kt} + E_t[sdf_{kt+1} V_{kt+1}]$, with the stochastic discount factor

$$sdf_{kt} = \left(1 + i_{kt}^{sd}\right) / \left(1 + \pi_{kt}^{C,vat}\right) \approx \left(1 + i_{kt-1}^{rf} + rprem_{kt-1}^{s}\right) / \left(1 + \pi_{kt}^{C,vat}\right)$$

which depends directly on the investment risk premium, $rprem_{kt-1}^{s}$. The dividends are defined as:

$$d_{ikt} = (1 - \tau_k^K) \left(\frac{P_{ikt}^{Y}}{P_{kt}^{Y}} Y_{ikt} - \frac{W_{kt}}{P_{kt}^{Y}} N_{ikt} \right) + \tau_k^K \delta \frac{P_{kt}^{I}}{P_{kt}^{Y}} K_{ikt-1} - \frac{P_{kt}^{I}}{P_{kt}^{Y}} I_{ikt} - adj_{ikt}$$

where I_{ikt} is physical investment, P_{kt}^{I} is investment price, τ_k^K is the profit tax, δ is capital depreciation rate and adj_{ikt} are adjustment costs associated with price P_{ikt}^{Y} and labour input N_{ikt} adjustment or moving capacity utilization cu_{ikt} and investment I_{ikt} away from their optimal level:

$adj_{ikt} = adj(P_{ikt}^{Y}) + adj(N_{ikt}) + adj(cu_{ikt}) + adj(I_{ikt})$ where

$$adj(P_{ikt}^{Y}) = \frac{\gamma^{p}}{2} Y_{kt} \left(\frac{P_{ikt}^{Y}}{P_{ikt-1}^{Y}} - 1 \right)^{2}$$

$$adj(N_{ikt}) = \frac{\gamma^n}{2} Y_{kt} \left(\frac{N_{ikt}}{N_{ikt-1}} - 1 \right)^2$$
$$adj(cu_{ikt}) = \frac{P_{kt}^I}{P_{kt}^Y} K_{ikt-1} \left(\gamma^{u,1}(cu_{ikt} - 1) + \frac{\gamma^{u,2}}{2} (cu_{ikt} - 1)^2 \right)$$
$$adj(I_{ikt}) = \frac{P_{kt}^I}{P_{kt}^Y} \left(\frac{\gamma^{I,1}}{2} K_{kt-1} \left(\frac{I_{ikt}}{K_{kt-1}} - \delta \right)^2 + \frac{\gamma^{I,2}}{2} \frac{(I_{ikt} - I_{ikt-1})^2}{K_{kt-1}} \right)$$

The maximization is subject to production function, standard capital accumulation equation:

$$K_{ikt} = (1 - \delta) K_{ikt-1} + I_{ikt}$$

and the usual demand condition which inversely links demand for variety *i* goods and the price of the variety:

$$Y_{ikt} = \left(\frac{P_{ikt}^Y}{P_{kt}^Y} \right)^{-\sigma^y} Y_{kt}$$

Let $adj_{X,ikt}$ for $X = P^Y, N, cu, I$ denote additional dynamic terms due to the existence of adjustment costs. Let also define $g_{kt}^X := \frac{X_{kt} - X_{kt-1}}{X_{kt-1}}$ the net growth rate of variable $X = N, Y, I, C, \ldots$ and $\pi_{kt}^X := \frac{\Delta P_{kt}^X}{P_{kt-1}^X}$ the inflation rate of a price deflator associated with variable $X = N, Y, I, C, \ldots$ The main optimality conditions of the differentiated goods producers are as follows.

The usual equality between the marginal product of labor and labor cost holds, with a wedge driven by the labor adjustment costs:

$$\mu_{kt}^y \alpha \frac{Y_{kt}}{N_{kt}} - adj_{N,ikt} = (1 - \tau^k) \frac{W_{kt}}{P_{kt}^Y}$$

with μ_{kt}^y being inversely related to the price mark-up. The capital optimality condition reflects the usual dynamic trade-off faced by the firm:

$$\frac{1 + \pi_{kt+1}^y}{1 + i_{kt+1}^{sd}} \frac{P_{kt+1}^I / P_{kt+1}^Y}{P_{kt}^I / P_{kt}^Y} \left(\mu_{kt+1}^y (1 - \alpha) \frac{P_{kt+1}^Y Y_{ikt+1}}{P_{kt+1}^I K_{ikt}^{tot}} + \tau^k \delta - adj_{kt}^{cu} / K_{ikt} + (1 - \delta) Q_{kt+1} \right) = Q_{kt}$$

where Q_{kt} has the usual Tobin's interpretation.

FOC w.r.t. investment implies that Tobin's Q varies due to the existence of investment adjustment costs:

$$Q_{kt} = 1 + adj_{I,ikt}$$

Firms adjust their capacity utilization depending on the conditions on the market via the optimality condition:

$$\frac{\mu_{kt}^{y}}{P_{kt}^{I}/P_{kt}^{Y}}(1-\alpha)\frac{Y_{kt}}{cu_{kt}} = adj_{cu,ikt}$$

Finally, the FOC w.r.t. differentiated output price pins down the price mark-up:

$$\frac{\sigma^{y}}{(\sigma^{y}-1)}\mu_{kt}^{y} = (1-\tau^{k}) + \frac{adj_{P^{Y},ikt}}{(\sigma^{y}-1)} + \varepsilon_{kt}^{\mu}$$

with ε_{kt}^{μ} being the markup shock. The latter equation, combined with the FOC w.r.t. labor implies the Phillips curve of the familiar form.

A.3. Trade

A.3.1. Import sector

Aggregate demand components

The final aggregate demand component goods C_{kt} (private consumption good), I_{kt}, (private investment good) G_{kt} (government consumption good) and I_{kt}^{G} (government investment good) are produced by perfectly competitive firms by combining domestic output, O_{kt}^{Z} with imported goods M_{kt}^{Z}, $Z = C, I, G, I^{G}$, using the following CES production function:

$$Z_{kt} = A_{kt}^{p^{z}}\left[(1-\varepsilon_{kt}^{M}s^{M,Z})^{\frac{1}{\sigma^{z}}}(O_{kt}^{Z})^{\frac{\sigma^{z}-1}{\sigma^{z}}} + (\varepsilon_{kt}^{M}s^{M,Z})^{\frac{1}{\sigma^{z}}}(M_{kt}^{Z})^{\frac{\sigma^{z}-1}{\sigma^{z}}}\right]^{\frac{\sigma^{z}}{\sigma^{z}-1}}$$

with $A_{kt}^{p^{z}}$ a shock to productivity in the sector producing goods Z and ε_{kt}^{M} is a shock to the share $s^{M,Z}$ of imports in domestic demand components. We assume that the log difference of the specific productivities, $A_{kt}^{p^{z}}$ is an AR(1), $\varepsilon_{kt}^{p^{z}}$ with mean $g^{p^{z}}$. It follows that the demand for the domestic and foreign part of demand aggregates is:

$$O_{kt}^{Z} = \left(A_{kt}^{p^{z}}\right)^{\sigma^{z}-1}(1-\varepsilon_{kt}^{M}s^{M,Z})\left(\frac{P_{kt}^{O}}{P_{kt}^{Z}}\right)^{-\sigma^{z}} Z_{kt}$$

$$M_{kt}^{Z} = \left(A_{kt}^{p^{z}}\right)^{\sigma^{z}-1}\varepsilon_{kt}^{M}s^{M,Z}\left(\frac{P_{kt}^{M}}{P_{kt}^{Z}}\right)^{-\sigma^{z}} Z_{kt}$$

where P_{kt}^{Z} are price deflators associated with Z_{kt}; they satisfy:

$$P_{kt}^{Z} = \left(A_{kt}^{p^{z}}\right)^{-1}\left[(1-\varepsilon_{kt}^{M}s^{M,Z})(P_{kt}^{O})^{1-\sigma^{z}} + \varepsilon_{kt}^{M}s^{M,Z}(P_{kt}^{M})^{1-\sigma^{z}}\right]^{\frac{1}{1-\sigma^{z}}}$$

Economy-specific final imports demand

Final imported goods are produced by perfectly competitive firms combining economy-specific homogenous imports goods, M_{lkt}, using CES production function:

$$M_{kt} = \left(\sum_l (s_{lkt}^M)^{\frac{1}{\sigma^{FM}}} (M_{lkt})^{\frac{\sigma^{FM}-1}{\sigma^{FM}}}\right)^{\frac{\sigma^{FM}}{\sigma^{FM}-1}}$$

where σ^{FM} is the price elasticity of demand for country *l*'s goods and $\sum_l s_{lkt}^M = 1$ are import shares. The demand for goods from country *l* is then:

$$M_{lkt} = s_{lkt}^M \left(\frac{P_{lkt}^M}{P_{kt}^M}\right)^{-\sigma^{FM}} M_{kt}$$

while the imports price:

$$P_{kt}^M = \left(\sum_l s_{lkt}^M (P_{lkt}^M)^{1-\sigma^{FM}}\right)^{\frac{1}{1-\sigma^{FM}}}$$

with P_{lkt}^M being the country-specific imports good prices.

Supply of economy- and sector-specific imports

The homogenous goods from country *l* are assembled by monopolistically competitive firms from economy- and sector- specific goods using a linear production function and subject to adjustment costs. All products from country *l* are initially purchased at export price P_{lt}^X of this country. Firms then maximize a discounted stream of profits, div_{kt}^{IM}, such that :

$$div_{ilkt}^{IM} = \frac{P_{ilkt}^M}{P_{kt}^Y} M_{ilkt} - e_{lkt} \frac{P_{lt}^X}{P_{kt}^Y} M_{ilkt} - adj_{ilkt}^{PM}$$

where adj_{ilkt}^{PM} are the adjustment costs that producers face when choosing the bilateral import price.[9] The maximization is subject to the usual inversely-sloping demand equation. These assumptions result in a simple expression for price P_{lkt}^M of homogenous goods from country *l*:

$$P_{lkt}^M = e_{lkt} P_{lt}^X - adj_{M,ilkt}^{PM}$$

where $adj_{M,ilkt}^{PM}$ are additional dynamic terms due to costs of adjustment.

A.3.2. Export sector

The exporting firms are supposed to be competitive and set their prices equal to the output price, up to a shock, ε_{kt}^X:

$$P_{kt}^X = \varepsilon_{kt}^X P_{kt}^O$$

[9] $adj_{ilkt}^{PM} = \frac{\gamma^{PM}}{2} \frac{P_{lkt}^M}{P_{kt}^Y} M_{lkt-1} \left(\frac{P_{ilkt}^M}{P_{ilkt-1}^M} - 1\right)^2$

A.4. EA and US policy

A.4.1. Monetary policy

Monetary policy is modelled by a Taylor rule where the ECB sets the policy rate i_{kt} in response to area-wide inflation and real GDP growth. The policy rate adjusts sluggishly to deviations of inflation and GDP growth from their respective target levels; it is also subject to random shocks:

$$i_{kt} - \bar{\imath} = \rho^{i}(i_{kt-1} - i) + (1 - \rho^{i})\left(\eta^{i\pi}\left(0.25\left(\sum_{r=0}^{3}\pi_{kt-r}^{c+g}\right) - \bar{\pi}^{C+G}\right) + \eta^{iy}(\tilde{y}_{kt})\right) + u_{kt}^{inom}$$

where $i = r + \pi^{C+G}$ is the steady state nominal interest rate, equal to the sum of the steady state real interest rate and CPI inflation and output gap $\tilde{y}_{kt} = log(Y_{kt}) - \bar{y}_{kt}$

where $\bar{y}_{kt} = \ln(Y_{kt}^{pot})$ with $Y_{kt}^{pot} = (\bar{A}_{kt}^{Y}\bar{N}_{kt})^{\alpha}(K_{ikt-1}^{tot})^{1-\alpha}$ is (log) potential output. Potential output at date t is the output level that would obtain if the labor input equaled state per capita house worked, date *t* capital stock were utilized at full capacity, and TFP at *t* equaled its trend component.

The Taylor rule may be extended to deal with economies with managed exchange rates and other exchange rate regimes, as in Vitek (2013).

It is assumed that the risk-free rate is equal to the policy rate: $i_{kt}^{sd} \equiv i_{kt}$.

A.4.2. Fiscal policy

Government expenditure and receipts can deviate temporarily from their long-run levels in systematic response to budgetary or business-cycle conditions and in response to idiosyncratic shocks. Concerning government consumption and government investment, we specify the following autoregressive equations:

$$\frac{G_{kt}}{\bar{Y}_{kt}A_{kt}^{PG}} - \bar{G} = \rho^{G}\left(\frac{G_{kt-1}}{\bar{Y}_{kt}A_{kt}^{PG}} - \bar{G}\right) + u_{kt}^{G}$$

$$\frac{I_{kt}^{G}}{\bar{Y}_{kt}A_{kt}^{PI}} - \bar{I}^{G} = \rho^{IG}\left(\frac{I_{kt-1}^{G}}{\bar{Y}_{kt}A_{kt}^{PI}} - \bar{I}^{G}\right) + u_{kt}^{IG}$$

$$\frac{T_{kt}}{\overline{\overline{P_{kt}^{Y}Y_{kt}}}} - \bar{T} = \rho^{T}\left(\frac{T_{kt-1}}{\overline{P_{kt}^{Y}}Y_{kt}} - \bar{T}\right) + \eta^{DEF,T}\left(\frac{\Delta B_{kt}^{gtot}}{P_{kt}^{Y}Y_{kt}} - def^{T}\right) + \eta^{B,T}\left(\frac{B_{kt}^{gtot}}{P_{kt}^{Y}Y_{kt}} - \bar{B}_{k}^{G}\right) + u_{kt}^{T}$$

with B_{kt}^{gtot} total nominal government debt. Government transfers react to the level of government debt and the government deficit relative to the associated debt and deficit targets $\bar{B}_{k}^{G}$ and def^{T}.

The government budget constraint is

$$B_{kt}^{g} = \left(1 + i_{kt-1}^{g}\right)B_{kt-1}^{g} - R_{kt}^{G} + P_{kt}^{G}G_{kt} + P_{kt}^{IG}I_{kt}^{G} + T_{kt}$$

where government (nominal) revenue:

$$R_{kt}^{G} = \tau_{k}^{K}(P_{kt}^{Y}Y_{kt} - W_{kt}N_{kt} - P_{kt}^{I}\delta_{k}K_{kt-1}) + \tau^{N}W_{kt}N_{kt} + \tau^{C}P_{kt}^{C}C_{kt} + tax_{kt}$$

consists of taxes on consumption, labor and corporate income as well as lump-sum tax.

Finally, the accumulation equation for government capital is:

$$K_{kt}^{G} = \left(1 - \delta^{G}\right)K_{kt-1}^{G} + I_{kt}^{G}$$

A.5. The RoW block

The model of the RoW economy (subscript k=RoW) is a simplified structure with fewer shocks. Specifically, the RoW consists of a budget constraint for the representative household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses labor as the sole factor input, and a New Keynesian Phillips curve. The RoW block abstracts from capital accumulation. There are shocks to labor productivity, price mark-ups, the subjective discount rate, the relative preference for domestic vs. imported goods, as well as monetary policy shocks in the RoW.

More specifically the budget constraint for the RoW representative household is:

$$P_{RoWt}^{Y}Y_{RoWt} + P_{RoWt}^{Oil}OIL_{RoWt} = P_{RoWt}^{C}C_{RoWt} + P_{RoWt}^{X}X_{RoWt} - \sum_{l}\frac{size_{l}}{size_{RoW}}e_{lRoWt}P_{lt}^{X}M_{lRoWt}$$

where X_{RoWt} are non-oil exports by the RoW, and the intertemporal equation for aggregate demand derived from the FOC for consumption:

$$\beta_{t}\frac{\lambda_{RoWt+1}}{\lambda_{RoWt}}\frac{1 + i_{RoWt}}{1 + \pi_{RoWt+1}^{C}} = 1$$

with $\beta_{t} = \beta\exp(\varepsilon_{RoWt}^{C})$, $\left(C_{RoWt} - hC_{RoWt-1}\right)^{-\theta} = \lambda_{RoWt}$ and ε_{RoWt}^{C} as the RoW demand shock. Note that

$$i_{RoWt} \equiv i_{RoWkt}^{g}$$

As for the EA and the US, final aggregate demand C_{RoWt} (in the absence of investment and government spending in the RoW block) is a combination of domestic output, Y_{RoWt} and imported goods, M_{RoWt}, using the following CES function:

$$C_{RoWt} = A_{RoWt}^{p}\left[(1 - \varepsilon_{RoWt}^{M}s^{M})^{\frac{1}{\sigma}}(Y_{RoWt}^{C})^{\frac{\sigma-1}{\sigma}} + (\varepsilon_{RoWt}^{M}s^{M})^{\frac{1}{\sigma}}(M_{RoWt}^{C})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

which gives the demand for the domestic and foreign goods in RoW demand:

$$Y^C_{RoWt} = \left(A^p_{RoWt}\right)^{\sigma-1}(1-\varepsilon^M_{RoWt}s^M)\left(\frac{P^Y_{RoWt}}{P^C_{RoWt}}\right)^{-\sigma} C_{RoWt}$$

$$M^C_{RoWt} = \left(A^p_{RoWt}\right)^{\sigma-1}\varepsilon^M_{RoWt}s^M\left(\frac{P^M_{RoWt}}{P^C_{RoWt}}\right)^{-\sigma} C_{RoWt}$$

where the consumer price deflator P^C_{RoWt} satisfies:

$$P^C_{RoWt} = \left(A^p_{RoWt}\right)^{-1}[(1-\varepsilon^M_{RoWt}s^M)(P^Y_{RoWt})^{1-\sigma} + \varepsilon^M_{RoWt}s^M(P^M_{RoWt})^{1-\sigma}]^{\frac{1}{1-\sigma}}$$

The RoW non-oil output is produced with the technology:

$$Y_{RoWt} = A^Y_{RoWt}N_{RoWt}$$

Price setting for RoW non-oil output follows a New Keynesian Phillips curve:

$$\pi^Y_{RoWt} - \bar{\pi}^Y_{RoW} = \beta\frac{\lambda_{RoWt+1}}{\lambda_{RoWt}}(sfp(E_t\pi^Y_{RoWt+1} - \bar{\pi}^Y_{RoW}) + (1-sfp)(\pi^Y_{RoWt-1} - \bar{\pi}^Y_{RoW})) + \varphi^Y_{RoW}\ln(Y_{RoWt} - \bar{Y}_{RoW}) + \varepsilon^Y_{RoWt}$$

Monetary policy in the RoW follows the Taylor rule:

$$i_{RoWt} - \bar{\imath} = \rho^i(i_{RoWt-1} - \bar{\imath}) + \left(1-\rho^i\right)\left(\eta^{i\pi}(\pi^Y_{RoWt} - \bar{\pi}^Y_{RoW}) + \eta^{iy}\tilde{y}_{RoWt}\right) + \varepsilon^{inom}_{RoWt}$$

where $\tilde{y}_{RoWt}$ is the deviation of actual output from trend output.

The RoW net foreign asset (NFA) position equals minus the sum of the EA and US NFA positions.

Finally, oil is assumed to be fully imported from the RoW and the oil price is assumed as follows:

$$P^{Oil}_{RoWt} = \frac{\bar{P}^Y}{A^{p^{oil}}_{ROWt}e_{RoW,US}}$$

where $A^{p^{oil}}_{ROWt}$ is oil-specific productivity and oil is priced in USD.

Total nominal exports are defined as:

$$P^X_{RoWt}X_{RoWt} = \sum_l P^X_{lRoWt}M_{RoWlt}$$

with the bilateral export price being defined as the domestic price subject to a bilateral price shock:

$$P^X_{lRoWt} = \exp(\varepsilon^{PX}_{lRoWt})\, P^Y_{RoWt}$$

A.6 Closing the economy

Market clearing requires that:

$$Y_{kt}P_{kt}^{Y} + div_{kt}^{M}P_{kt}^{Y} = P_{kt}^{C}C_{kt} + P_{kt}^{I}I_{kt} + P_{kt}^{IG}IG_{kt} + TB_{kt}$$

Export is a sum of imports from the domestic economy by other countries:

$$X_{kt} = \sum_{l} M_{klt}$$

where M_{klt} stands for imports from the domestic economy to economy *l*. The total imports are defined as:

$$P_{kt}^{Mtot}M_{kt}^{tot} = P_{kt}^{M}M_{kt} + P_{kt}^{oil}OIL_{kt}$$

where non-oil imports

$$P_{kt}^{M}M_{kt} = P_{kt}^{M}(M_{kt}^{C} + M_{kt}^{I} + M_{kt}^{G} + M_{kt}^{IG})$$

Net foreign assets, NFA_{kt}, evolve according to

$$e_{RoWk,t}B_{k,t}^{w} = +(1 + i_{t-1}^{bw})e_{RoWk,t}B_{k,t-1}^{w} + P_{kt}^{X}X_{kt} - \sum_{l} \frac{size_l}{size_k} e_{lkt}P_{lt}^{X}M_{lkt} - P_{kt}^{Oil}OIL_{kt} + ITR_k\overline{P_{kt}^{Y}Y_{kt}}$$

where $P_{kt}^{X}X_{kt} - \sum_l \frac{size_l}{size_k} e_{lkt}P_{lt}^{X}M_{lkt} - P_{kt}^{Oil}OIL_{kt} = TB_{kt}$ defines the trade balance, with domestic importers buying the imported good at the price $\mathrm{P}_{\mathrm{lt}}^{\mathrm{X}}$ .We allow non-zero trade balance and include an international transfer, ITR_k, calibrated in order to satisfy zero NFA in equilibrium.

Finally, net foreign assets of each country sum to zero:

$$\sum_{l} NFA_{lt}size_l = 0.$$

$size_l$ is the relative size of economy *l*.

Table A.1. Prior and posterior distributions of key estimated model parameters: EA and US

	Posterior distributions				Prior distributions		
	EA		US				
	Mode	Std	Mode	Std	Distrib.	Mean	Std
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Preferences							
h	0.88	0.07	0.86	0.06	B	0.5	0.2
h_N	0.38	3.13	0.85	0.10	B	0.5	0.2
θ	1.47	0.87	1.37	0.61	G	1.5	0.2
θ^N	2.32	4.91	2.10	3.74	G	2.5	0.5
α_{RoW}^{b1}	0.007	0.005	0.002	0.004	B	0.005	0.005
Steady state share of Ricardian households							
ω^s	0.70	0.03	0.74	0.04	B	0.65	0.05
Production functions							
σ^o	0.33	0.19	0.33	0.13	B	0.5	0.08
σ^z	4.30	0.71	4.05	0.76	G	2	1
σ^{FM}	1.06	0.94	0.28	0.61	G	2	1
Nominal and real frictions							
γ^p	23.2	10.1	57.4	36.8	G	60	40
SFP	0.60	0.09	0.75	0.11	B	0.5	0.2
γ^w	4.69	8.22	2.93	4.71	G	5	2
γ^{wr}	0.97	0.06	0.96	0.05	B	0.5	0.2
SFW	0.53	0.31	0.51	0.62	B	0.5	0.2
$\gamma^{I,1}$	8.79	7.16	16.2	8.11	G	60	40
$\gamma^{I,2}$	26.6	13.8	12.2	7.79	G	60	40
γ^n	4.90	1.80	12.8	8.06	G	60	40
$\gamma^{u,2}$	0.06	0.03	0.07	0.05	B	0.1	0.04
γ^{pM}	0.13	0.09	0.40	0.21	B	2	0.8
Monetary policy							
ρ^i	0.85	0.03	0.83	0.03	B	0.7	0.12
$\eta^{i\pi}$	2.23	0.56	1.76	0.24	B	2	0.4
η^{iy}	0.08	0.05	0.07	0.02	B	0.5	0.2
Fiscal policy							
ρ^T	0.97	0.05	0.96	0.07	B	0.7	0.1
$\eta^{DEF,T}$	-0.01	0.00	-0.01	0.01	B	-0.03	0.01
$\eta^{B,T}$	-0.001	0.00	-0.001	0.00	B	-0.001	0.001
ρ^G	0.95	0.01	0.95	0.02	B	0.7	0.1
ρ^{IG}	0.85	0.07	0.92	0.04	B	0.7	0.1
Autocorrelations of forcing variables							
ρ^{AY}	0.78	0.12	0.50	0.16	B	0.5	0.2
$\rho^{\overline{AY}}$	0.95	0.12	0.96	0.04	B	0.85	0.075
$\rho^{Ap,C}$	0.08	0.15	0.34	0.12	B	0.5	0.2
$\rho^{Ap,G}$	0.20	0.09	0.27	0.14	B	0.5	0.2
$\rho^{Ap,I}$	0.15	0.17	0.39	0.13	B	0.5	0.2
$\rho^{Ap,IG}$	0.88	0.04	0.97	0.01	B	0.5	0.2
ρ^c	0.80	0.10	0.79	0.08	B	0.5	0.2
ρ^b	0.91	0.05	0.93	0.05	B	0.5	0.2
$\rho^{b,RoW}$	0.85	0.09	0.57	0.46	B	0.5	0.2

ρ^{s}	0.94	0.05	0.96	0.03	B	0.85	0.05
ρ^{μ}	0.24	0.15	0.76	0.12	B	0.5	0.2
ρ^{M}	0.93	0.02	0.85	0.05	B	0.5	0.2
ρ^{X}	0.94	0.02	0.85	0.04	B	0.5	0.2
ρ^{PX}_{lRoW}	0.98	0.01	0.98	0.01	B	0.5	0.2
Standard deviations (%) of innovations to forcing variables							
u^{AY}	0.92	0.17	1.45	0.35	G	0.50	0.20
$u^{L\overline{AY}}$	0.09	1.41	0.09	0.98	G	0.10	0.04
$u^{G\overline{AY}}$	0.02	0.05	0.03	0.03	G	0.02	0.008
$u^{Ap,C}$	0.19	0.02	0.20	0.03	G	1.00	0.40
$u^{Ap,G}$	0.45	0.04	0.39	0.04	G	1.00	0.40
$u^{Ap,I}$	0.32	0.05	0.59	0.05	G	1.00	0.40
$u^{Ap,IG}$	0.76	0.05	0.38	0.05	G	1.00	0.40
u^{c}	0.96	0.92	1.19	0.91	G	1.00	0.40
u^{b}	0.11	0.01	0.13	0.01	G	1.00	0.40
$u^{b,RoW}$	0.24	0.01	0.10	0.27	G	1.00	0.40
u^{s}	0.18	0.09	0.16	0.08	G	0.10	0.02
u^{μ}	3.65	1.55	3.80	2.93	G	2.00	0.80
u^{n}	1.16	1.87	1.84	2.35	G	1.00	0.40
u^{M}	5.83	0.91	6.51	1.07	G	2.00	0.80
u^{X}	0.61	0.07	0.81	0.04	G	1.00	0.40
u^{PX}_{lRoW}	4.24	0.47	2.45	0.18	G	1.00	0.40
u^{i}	0.09	0.01	0.12	0.01	G	1.00	0.40
u^{G}	0.06	0.01	0.13	0.01	G	1.00	0.40
u^{IG}	0.12	0.01	0.07	0.01	G	1.00	0.40
u^{T}	0.10	0.01	0.23	0.02	G	1.00	0.40

Notes: Cols. (1) lists model parameters. Cols. (2)-(3) and Cols. (4)-(5) show the mode and the standard deviation (Std) of the posterior distributions of EA parameters and of US parameters, respectively. Cols. (6) (labelled 'Distrib.') indicates the prior distribution function (B: Beta distribution; G: Gamma distribution). Identical priors are assumed for EA and US parameters.

Table A.2. Prior and posterior distributions of key estimated model parameters: ROW

	Posterior distributions		Prior distributions		
	Mode	Std	Distribution	Mean	Std
(1)	(2)	(3)	(4)	(5)	(6)
Preferences					
h	0.95	0.03	B	0.7	0.1
θ	1.48	1.53	G	1.5	0.2
α_{RoW}^{b1}	0.007	0.005	B	0.002	0.0008
Firms					
σ	0.06	0.21	G	2	1
SFP	0.96	0.13	B	0.5	0.2
φ_{RoW}^{Y}	0.11	0.04	G	0.5	0.2
Monetary policy					
ρ^{i}	0.93	0.01	B	0.7	0.1
$\eta^{i\pi}$	1.03	0.16	B	2	0.4
η^{iy}	1.06	0.43	B	0.5	0.2
Autocorrelations of forcing variables					
$\rho^{\overline{AY}}$	0.90	0.07	B	0.85	0.075
ρ^{c}	0.81	0.09	B	0.5	0.2
ρ^{M}	0.85	0.09	B	0.5	0.2
ρ^{Y}	0.80	0.08	B	0.5	0.2
Standard deviations (%) of innovations to forcing variables					
$u^{G\overline{AY}}$	0.39	0.08	G	0.1	0.04
u^{c}	1.21	1.12	G	1.00	0.40
u^{M}	3.12	1.01	G	2.00	0.80
u^{Y}	0.08	0.03	G	1.00	0.40
u^{i}	0.07	0.01	G	1.00	0.40

Notes: Cols. (1) lists model parameters. Cols. (2)-(3) show the mode and the standard deviation (Std) of the posterior distributions of EA parameters and of US parameters, respectively. Cols. (4) indicates the prior distribution function (B: Beta distribution; G: Gamma distribution).

Table A.3. Calibrated model parameters and ratios

		EA	US	RoW
(1)	(2)	(3)	(4)	(5)
Preferences				
Intertemporal discount factor	β	0.9990	0.9986	0.9992
Import share in final demand	s^{M}	0.16	0.12	0.09
Preference for imports from RoW	$s^{M,ROW}$	0.86	0.79	
Preference for imports from US	$s^{M,US}$	0.14		0.50
Preference for imports from EA	$s^{M,EA}$		0.21	0.50
Substitutability btw domestic varieties	σ^{y}	5.1	18.9	
Preference for gov bonds	α^{b0}	-0.002	-0.002	
Preference for stocks	α^{s0}	-0.002	0.002	
Preference for foreign bonds	α^{bw0}	0.012	0.012	
Weight of disutility of labour	ω^{N}	26.0	436.6	
Production				
Cobb-Douglas labour share	α	0.65	0.65	
Depreciation of private capital stock	δ	0.014	0.017	
Depreciation of public capital stock	δ^{G}	0.014	0.017	
Share of oil in total output	s^{Oil}	0.03	0.03	
Linear capacity utilisation adj costs	$\gamma^{u,1}$	0.02	0.02	
Monetary policy				
Nominal interest rate in SS	i	0.008	0.008	0.008
CPI inflation in SS	π^{C}	0.004	0.005	
GDP inflation in SS (p.a.)	π^{Y}			0.02
Persistence in Taylor rule	ρ^{i}			0.50
Fiscal policy				
Gov consumption share in SS	G/Y	0.22	0.17	
Gov investment share in SS	IG/Y	0.03	0.04	
Transfers share in SS	T/Y	0.18	0.10	
Consumption tax	τ^{C}	0.20	0.20	
Corporate profit tax	τ^{K}	0.30	0.30	
Labour tax	τ^{N}	0.45	0.21	
Deficit target	def^{T}	0.024	0.025	
Debt target	B^{G}	3.2	3.4	
Size of the region (% of world)	size	17	25	58
Trend of total factor productivity	g^{AY}	0.0022	0.0023	0.0025
Trend of private consumption specific productivity	g^{PC}	-0.0002	-0.0006	
Trend of gov consumption specific productivity	g^{PG}	-0.0005	-0.0038	
Trend of private investment specific productivity	g^{PI}	0.0006	0.0005	
Trend of gov investment specific productivity	g^{PIG}	0.0006	0.0005	

Notes: Col. (1) lists model parameters, Col. (2) the corresponding symbols in the model description, and Cols. (3)-(5) the respective parameter values in the EA, US (and if applicable) RoW blocks.

B. Construction of Rest-of-World (ROW) aggregates

The series for ROW real GDP (GDPR) is constructed as follows. First, we normalise the series for GDP in national currency (NAC) at constant prices for each country (i) at the common base year t=0:

$$\frac{GDPR_t^i}{GDPR_0^i} = \prod_{k=1}^{t} \left(\frac{GDPR_k^i}{GDPR_{k-1}^i}\right)$$

Then we calculate the time-varying share of each country in the block based on nominal GDP (GDPN) in USD. Finally, we compute ROW GDPR as the GDPN-weighted average of the 58 countries, which gives the ROW GDPR index with base year t=0:

$$GDPR_t^{ROW} = \sum_{i=1}^{58} \frac{GDPN_t^{USD,i}}{GDPN_t^{USD,ROW}} GDPR_t^i$$

The aggregation applies time-varying weights in order to account for changes in the relative economic weight of individual ROW countries over the sample period. ROW GDPR is normalised to 1 in year 2005.

The series for the ROW GDP deflator (PGDP) is constructed analogously to the ROW GDPR series. First, we normalise the series for the PGDP for each country (i) to base year t=0:

$$\frac{PGDP_t^i}{PGDP_0^i} = \prod_{k=1}^{t} \left(\frac{PGDP_k^i}{PGDP_{k-1}^i}\right)$$

Then we calculate the time-varying share of each country in the block based on GDP in USD and compute the ROW PGDP as the GDP-weighted average of the 58 country series, which gives the ROW GDPR index with base year t=0:

$$PGDP_t^{ROW} = \sum_{i=1}^{58} \frac{GDPN_t^{USD,i}}{GDPN_t^{USD,ROW}} PGDP_t^i$$

ROW GDPR is normalised to 1 in year 2005. An index of ROW nominal GDP (GDPN) with base year 2005 can be calculated by multiplying ROW GDPR with ROW PGDP.

The ROW block in the model has a flexible nominal exchange rate. The ROW nominal exchange rate to the USD (e) is calculated as GDP-weighted average of bilateral exchange rates against the USD for the 58 countries. As for GDPR and PGDP above, we normalise bilateral USD exchange rates in each country to the base year t=0:

$$\frac{e_t^{i,\$}}{e_0^{i,\$}} = \prod_{k=1}^{t} \left(\frac{e_k^{i,\$}}{e_{k-1}^{i,\$}}\right)$$

The ROW nominal exchange rate to the USD with base year t=0 is then calculated as GDP-weighted average of the 58 country series:

$$e_t^{ROW,\$} = \sum_{i=1}^{58} \frac{GDPN_t^{i,\$}}{GDPN_t^{ROW,\$}} e_t^{i,\$}$$

The ROW exchange rate to the USD is normalised to 1 in 2005. The exchange rate series includes exchange rate movements between members of the ROW group instead of attributing them to the ROW price index.

The short-term interest rate for the ROW is the GDP-weighted average of interest rate series for countries (i) in the ROW. The sample is reduced to 47 countries due to limited data availability and the GDP weights are adjusted accordingly.

The ROW trade balance (TB) balances international trade flows:

$$TB_t^{ROW} = -(TB_t^{EA} + TB_t^{US})$$

ROW exports equal the sum of EA and US imports from the ROW. The bilateral imports from the ROW are obtained by subtracting imports from the US (EA) from total EA (US) imports based on trade matrices for international good and service trade. Analogously, imports of the ROW equal EA plus US exports to the ROW.

C. Model-predicted and empirical business cycle statistics (first-differenced variables)

	Model				Data			
variable	**std (%)**	**Corr. with domestic GDP**	**Cross-country corr. with EA**	**Cross-country corr. with US**	**std (%)**	**Corr. with domestic GDP**	**Cross-country corr. with EA**	**Cross-country corr. with US**
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
GDP EA	0.62	1.00		0.27	0.64	1.00		0.52
Consumption EA	0.53	0.27			0.39	0.69		
Investment EA	3.20	0.60			2.77	0.80		
Government cons. EA	0.35	0.03			0.34	0.00		
Hours worked EA	0.56	0.62			0.49	0.74		
Interest rate EA	0.52	0.08			0.40	0.14		
GDP deflator EA	0.38	-0.11		0.19	0.20	0.06		0.36
Consumption price EA	0.37	0.21			0.31	0.57		
Exchange rate EA/US	4.06	0.15			4.19	0.10		
Trade balance/GDP EA	1.92	-0.05			0.92	-0.12		
GDP US	0.73	1.00	0.27		0.67	1.00	0.52	
Consumption US	0.70	0.40			0.59	0.67		
Investment US	3.36	0.59			3.30	0.79		
Government cons. US	0.86	0.13			0.83	0.11		
Hours worked US	0.56	0.53			0.58	0.48		
Interest rate US	0.60	0.02			0.55	-0.01		
GDP deflator US	0.47	-0.12	0.19		0.25	0.22	0.36	
Consumption price US	0.56	0.10			0.38	0.35		
Trade balance/GDP US	2.48	-0.03			1.120	-0.07		
GDP ROW	0.96	1.00	-0.05	0.06	0.85	1.00	0.19	0.43
GDP deflator ROW	0.90	-0.43	0.12	0.14	0.85	0.84	-0.09	0.36
Exchange rate ROW_US	1.91	0.05			1.41	-0.18		

Note: the Table reports model predicted standard deviations (Col.,1), correlations with domestic GDP (Col. 2) and cross country correlations for GDP and GDP deflators (Cols. 3-4), as well as the corresponding empirical statistics based on quarterly data for the period 1999q1-2014q4 (Cols. (5)-(8)). All statistics pertain to growth rates (first differences for interest rates and the trade balance/GDP ratio). The model-predicted moments are generated by a version of the linearized model in which the covariance matrix of all exogenous variables is set at the covariance matrix of the smoothed estimates of the innovations.

D. Fiscal policy under the ZLB constraint

We perform smoothed estimates of latent variables and shocks in the DSGE model by enforcing the ZLB constraints in both EA and US, using the estimated parameters in the baseline estimation without ZLB. We use the Occbin solution method developed by Guerrieri and Iacoviello (2015) to treat the occasionally binding constraint via a piecewise linear solution[10]. Moreover, we implement an algorithm similar to Anzoategui, et al. (mimeo, 2015, Appendix A2) to obtain smoothed estimates of latent variables as well the sequence of regimes along the historical periods. Finally, we use these smoothed estimates to measure the *non-additive* impact of individual shocks onto GDP, namely a non-linear extension of linear/additive historical shock decompositions (M. Ratto, 2016).

D.1 Estimation of latent variables and shocks under the ZLB

Let i_{kt}^{NC} be the unconstrained nominal interest rate that follows the Taylor rule without monetary shock:

$$i_{kt} - \bar{\imath} = \rho^i (i_{kt-1} - i) + (1 - \rho^i) \left(\eta^{i\pi} \left(0.25 \left(\sum_{r=0}^{3} \pi_{kt-r}^{c+g} \right) - \bar{\pi}^{C+G} \right) + \eta^{iy} (\tilde{y}_{kt}) \right) + u_{kt}^{inom}$$

The actual realized nominal policy interest rate i_{kt} set by the central bank will follow the usual Taylor rule if $i_{kt}^{NC} > i^{LB}$:

$$i_{kt} = i_{kt}^{NC} + \upsilon_{kt}^{inom}$$

while it will be constrained if $i_{kt}^{NC} \leq i^{LB}$

$$i_{kt} = i^{LB} + \upsilon_{kt}^{inom}$$

We set the lower bound for quarterly short-term nominal interest rates at 0.001 (i.e. 0.4% yearly). Under the constrained ZLB regime, the variable i_{kt}^{NC} acts as a 'shadow' interest rate that, within the Occbin algorithm, allows determining endogenously when the constraint is no longer binding. Moreover, we still use an exogenous monetary shock under the constrained regime, in order to keep observing the actual policy rates in the data. This shock does not alter the behavior of the piecewise linear solution in terms of transmission mechanisms under the ZLB constraint.

The algorithm for estimating latent variables is as follows:

1) Guess an initial sequence of regimes for each historical period $R_t^{(0)}$ for $t = 1, \dots T$
2) Given the sequence of regimes, compute the piecewise linear state space matrices $\boldsymbol{\Upsilon}_{\boldsymbol{t}}^{(0)}$ following the Occbin methodology
3) For each iteration $j = 1, \dots, n$

[10] This solution may also be viewed as an iterative application of the solution of Cagliarini and Kulish (2013) where the iterations are used to find the expected duration, which is consistent with the binding constraint.

a. feed the state space matrices $\boldsymbol{\Upsilon}_t^{(j-1)}$ to a Kalman Filter[11]/ Fixed interval smoothing algorithm to determine initial conditions, smoothed variables $\boldsymbol{y}_t^{(j)}$ and shocks $\boldsymbol{\epsilon}_t^{(j)}$.
b. given initial conditions and shocks perform Occbin simulations that endogenously determine a new sequence of regimes $R_t^{(j)}$, from which a new sequence of states space matrices is derived $\boldsymbol{\Upsilon}_t^{(j)}$

4) The algorithm stops when $R_t^{(j)} = R_t^{(j-1)}$ for all $t = 1, \dots, T$.

The algorithm used for estimating latent variables yields initial conditions and a sequence of smoothed variables and shocks, consistent with the observables, and taking into account the occasionally binding constraint, i.e. it also estimates a sequence of regimes along the historical periods.

Table 1 Estimation of the historical sequence of occasionally binding regimes

	EA		*US*	
time	regime sequence[12]	starting period of regime[13]	regime sequence	starting period of regime
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0	1
2009	0 1 0	1 3 8	0 1 0	1 3 7
2009.25	0 1 0	1 2 7	0 1 0	1 2 7
2009.5	0 1 0	1 2 4	1 0	1 3
2009.75	1 0	1 2	1 0	1 2
2010	0	1	0	1
2010.25	0	1	0	1
2010.5	0	1	0	1
2010.75	0	1	0	1
2011	0	1	0	1
2011.25	0	1	0	1
2011.5	0	1	0	1
2011.75	0	1	0	1
2012	0	1	0	1
2012.25	0	1	0	1
2012.5	0	1	0	1
2012.75	0	1	0	1

[11] Kulish et al. (2014) also apply the piecewise linear solution in the Kalman filter to estimate DSGE models with forward guidance.
[12] 0 = unconstrained; 1 = constrained.
[1 0] indicates a constrained regime. [0 1 0] indicates a regime that anticipates FUTURE constraints.
[13] Periods for which the regime starts.
[1 7] indicates a constrained regime for 6 periods. [1 2 7] indicates a regime that anticipates FUTURE constraints starting in period 2 until period 6.

2013	1 0	1 3	1 0	1 2
2013.25	1 0	1 3	1 0	1 3
2013.5	1 0	1 3	1 0	1 3
2013.75	1 0	1 3	1 0	1 3
2014	1 0	1 3	1 0	1 3
2014.25	1 0	1 3	0	1
2014.5	1 0	1 3	0	1
2014.75	1 0	1 3	1 0	1 3

The sequence of regimes is reported in the next Table 1. It is worth noting that agents in both EA and US anticipated ZLB starting in 2009q1. EA in particular anticipates quite prolonged ZLB, which influences significantly shock contributions in EA in 2009. Moreover, *both* EA and US faced a constrained monetary policy in the second half of 2009. Monetary policy is again constrained for both US and EA since 2013q1.

In Figure 1 we show the historical pattern of the 'shadow' unconstrained nominal interest rate (i_{kt}^{NC}, black dots) versus the actual data of policy rates (i_{kt}, red)[14]. When i_{kt}^{NC} is below the threshold the constraint is binding, otherwise the regime will be either 'normal' or anticipating future binding regimes. This is provided in Table 1 as well.

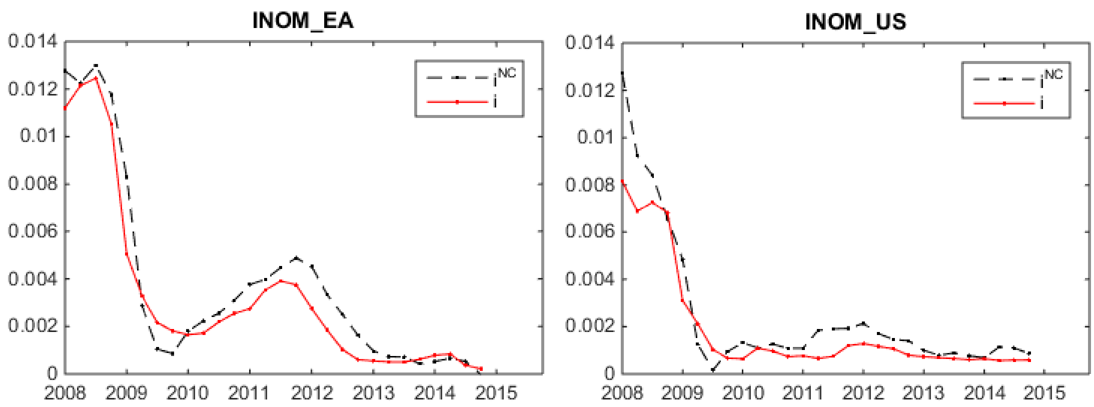


Figure 1. Observed interest rate vs. 'shadow' interest rate

D.2 Estimating contributions of shocks for the piecewise linear solution

One interesting issue is the estimation of the shock contributions to the observed data consistent with the piecewise linear solution, namely the extension of the standard historical shock decompositions to the case of occasionally binding regimes. The contribution of individual smoothed shocks, however, is not the mere additive superposition of each shock propagated by the sequence of state space matrices $\mathbf{\Upsilon}^{(j)}$ estimated with the smoother. The sequence of regimes associated to the state matrices, in fact, is a *non-linear function* of the whole set of shocks *simultaneously* affecting the economy, i.e. it is *conditional* on the sequence and combination of shocks simultaneously hitting the economy:

$$\mathbf{\Upsilon}_{\boldsymbol{t}}^{(\epsilon)} = f(\epsilon_{1t}, \dots, \epsilon_{kt}), t = 1, \dots, T$$

[14] Note that, in the estimation, we used the money market rate for US. The latter fell less abruptly in 2008/09 than the Fed Funds rate.

It is easy to verify that, taking subsets of shocks or individual shocks, the sequence of regimes will change. One way of measuring the effect of shocks in this non-linear context is to consider simulations conditional to given shock patterns.

In particular, we consider a definition that generalize the concept of shock contributions to the non-linear case, which degenerates to the standard shock decompositions for the linear case (M. Ratto, 2016). We define ϵ_{lt} the shock or group of shocks of interest, while $\epsilon_{\sim lt}$ denotes the complementary set of shocks. We compute the contribution of ϵ_{lt} by setting to zero the shocks ϵ_{lt} and performing simulations using the initial condition *and* the sequence of smoothed shocks for the *complementary* set of shocks $\epsilon_{\sim lt}$. We define this simulation as $y_t(\epsilon_{\sim lt}|\epsilon_{\sim lt}, y_0)$. The contribution of the shocks of interest will be the *complement* of this simulation to the smoothed variable y_t : $y_t(\epsilon_{lt}|\epsilon_{\sim lt}, y_0) = y_t - y_t(\epsilon_{\sim lt}|\epsilon_{\sim lt}, y_0)$. We call this the *residual contribution* of ϵ_{lt}.

Note that each of these simulations provides a different sequence of regimes, which in general will be different from the historical one. We use the *residual contribution* to measure the impact of the ZLB on the contribution of shocks to observed variables. In particular, we focus in Figure 2 on the impact of fiscal shocks on yoy GDP in EA and US.

The two major outcomes of this non-linear analysis are:

a) the effect of fiscal shocks in 2009 changes *both in sign and size*, implying a significant *positive contribution* of fiscal measures at the onset of the great recession;
b) the negative contribution of fiscal shocks in the subsequent slump is magnified by the ZLB. This makes the impact of fiscal policy more visible, although this is still not the main driver of the slump. In EA, in particular, the contribution of fiscal shocks in 2013 is about -0.35% out of a maximum decline of about -2.6% in 2013q1 (i.e. about 15% the decline). In 2014, the fiscal shocks under ZLB still have a negative impact in the first three quarters, by about -0.15% out of an overall GDP decline of -0.55%. The linear shock decomposition, in turn, implies no or slightly positive contribution of the fiscal shocks in 2014 for the EA.

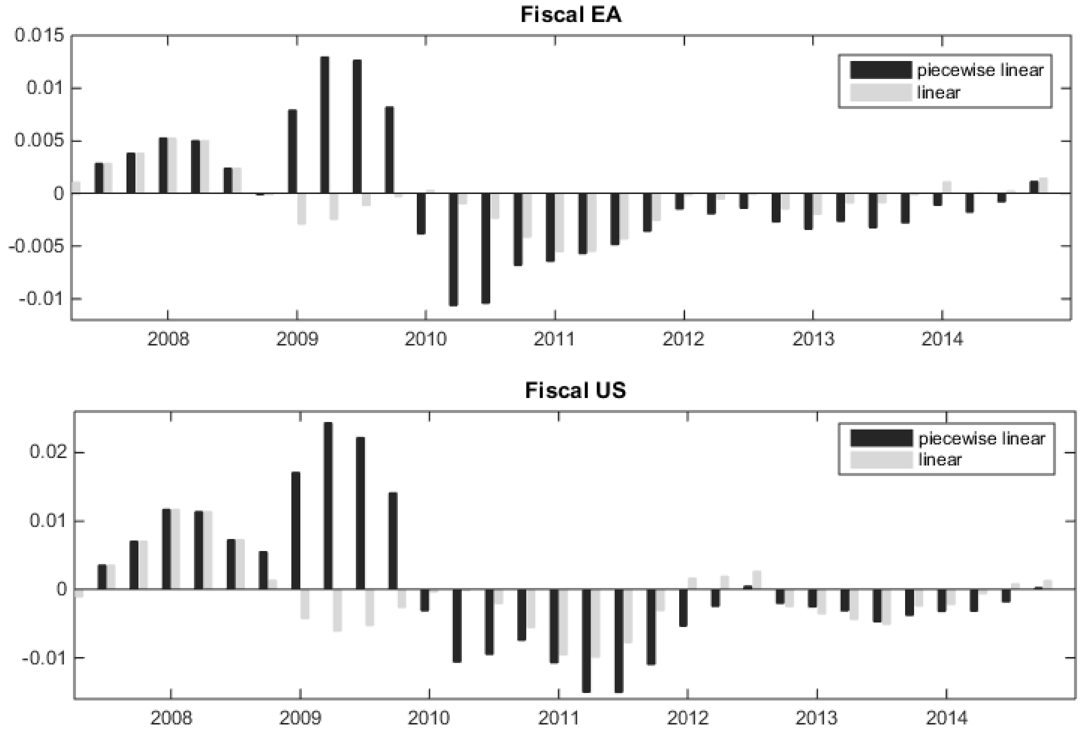


Figure 2. Contribution of fiscal shocks to yoy GDP growth in EA and US. Comparison of linear and piecewise linear solutions

To better understand the interaction between shocks and regime sequences, we report in Table 2 the regimes obtained shutting off the fiscal shocks. This shows that, for both EA and US, without fiscal shocks there would have been more severely binding constrained regimes in 2009. Moreover, in EA, constrained regimes would be less binding in 2013 without fiscal shocks. In US, the absence of fiscal shocks would have implied more prolonged constrained regimes in 2010, 2012 and 2014.

Table 2 Sequence of regimes obtained removing fiscal shocks in EA and US respectively

	EA		*US*	
time	regime sequence[15]	starting period of regime[16]	regime sequence	starting period of regime
2008	0	1	0	1
2008.25	0	1	0	1
2008.5	0	1	0	1
2008.75	0	1	0 1 0	1 3 6
2009	0 1 0	1 2 8	1 0	1 9
2009.25	1 0	1 7	1 0	1 8
2009.5	1 0	1 5	1 0	1 6

[15] 0 = unconstrained; 1 = constrained.
[1 0] indicates a constrained regime. [0 1 0] indicates a regime that anticipates FUTURE constraints.
[16] Periods for which the regime starts.
[1 7] indicates a constrained regime for 6 periods. [1 2 7] indicates a regime that anticipates FUTURE constraints starting in period 2 until period 6.

2009.75	1 0	1 3	1 0	1 4
2010	1 0	1 3	1 0	1 3
2010.25	0	1	1 0	1 5
2010.5	0	1	1 0	1 4
2010.75	0	1	1 0	1 3
2011	0	1	1 0	1 3
2011.25	0	1	0	1
2011.5	0	1	0	1
2011.75	0	1	0	1
2012	0	1	0 1 0	1 2 3
2012.25	0	1	1 0	1 3
2012.5	0	1	1 0	1 4
2012.75	0	1	1 0	1 5
2013	0	1	1 0	1 3
2013.25	0	1	1 0	1 4
2013.5	0 1 0	1 2 3	1 0	1 3
2013.75	1 0	1 3	1 0	1 3
2014	1 0	1 2	1 0	1 5
2014.25	1 0	1 3	1 0	1 3
2014.5	1 0	1 3	1 0	1 3
2014.75	1 0	1 4	1 0	1 4

We repeat the same exercise for the investment risk premia shocks. As shown in Figure 3, the effect of these shocks is (slightly) amplified considering the ZLB constraint. Moreover, looking at Figure 4, we can also note that this amplified effect is obtained with *smaller values* of the smoothed shock estimated with the piecewise linear solution during the ZLB regimes. Therefore, the transmission mechanism itself triggers this amplification.

Finally, it is worth mentioning that we obtained *unconstrained* monetary policy for the entire historical period for EA (US), when the investment risk premium shock in EA (US) is set to zero.

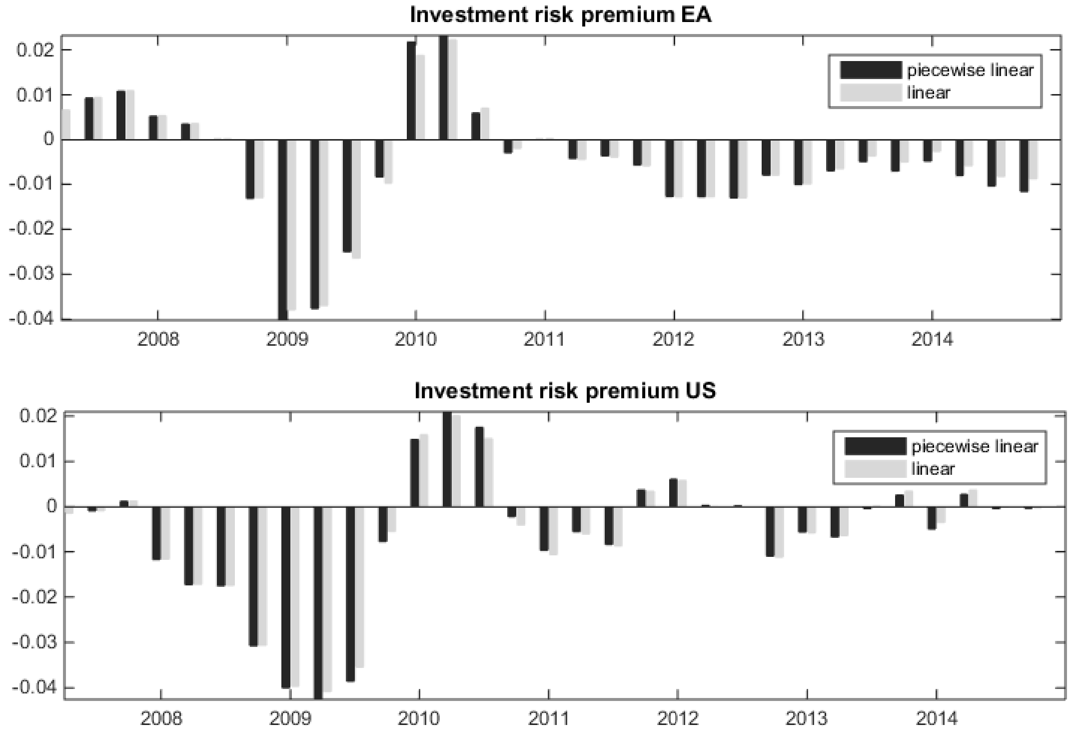


Figure 3. Contribution of investment risk premia shocks to yoy GDP growth in EA and US. Comparison of linear and piecewise linear solution

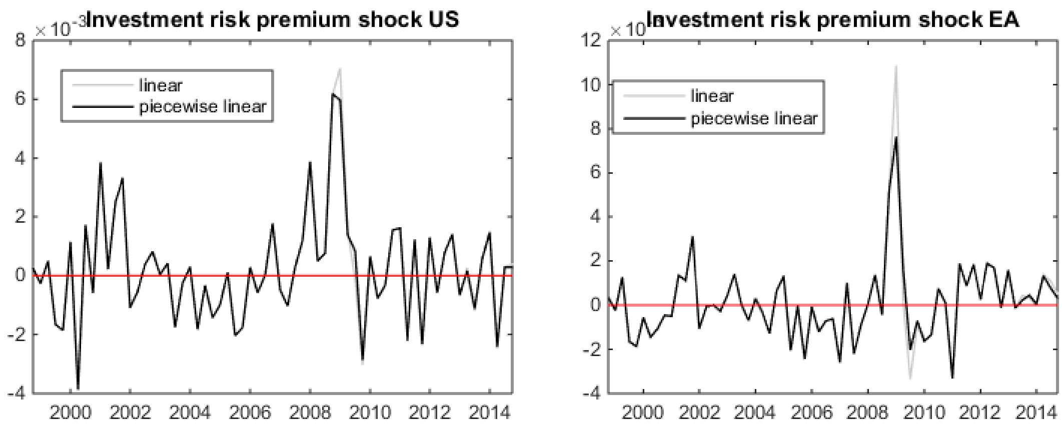


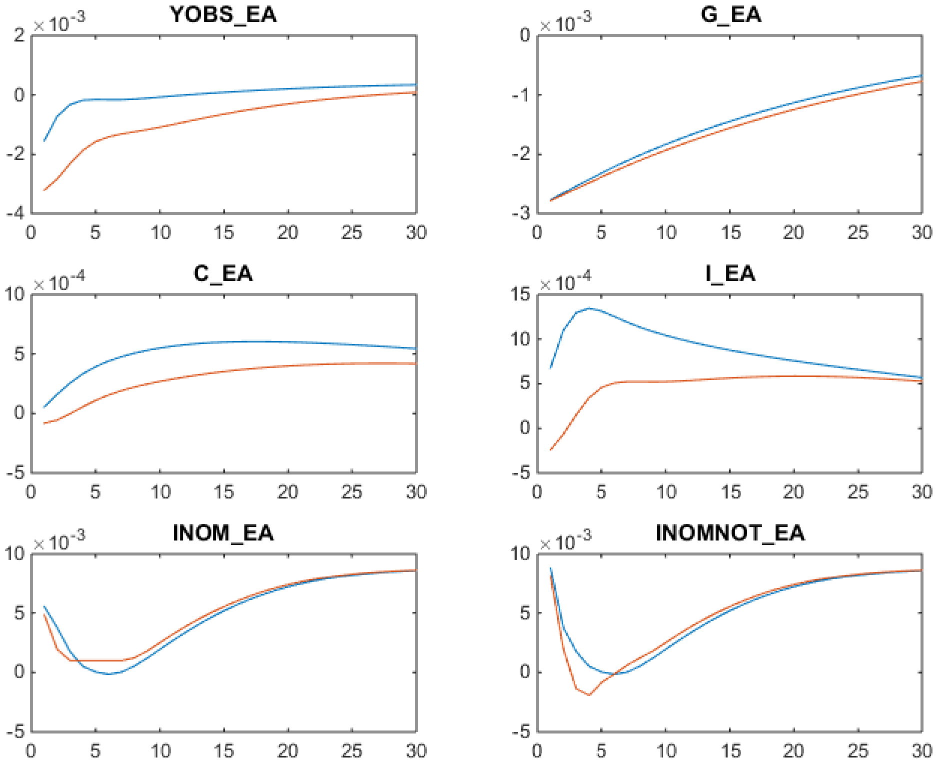
Figure 4. Smoothed estimate of innovations to the investment risk premia AR(1) processes.

D.3 Fiscal multipliers/IRFs under constrained regimes

We perform IRFs with ZLB consistent with the estimated timing and duration of the constrained regimes.

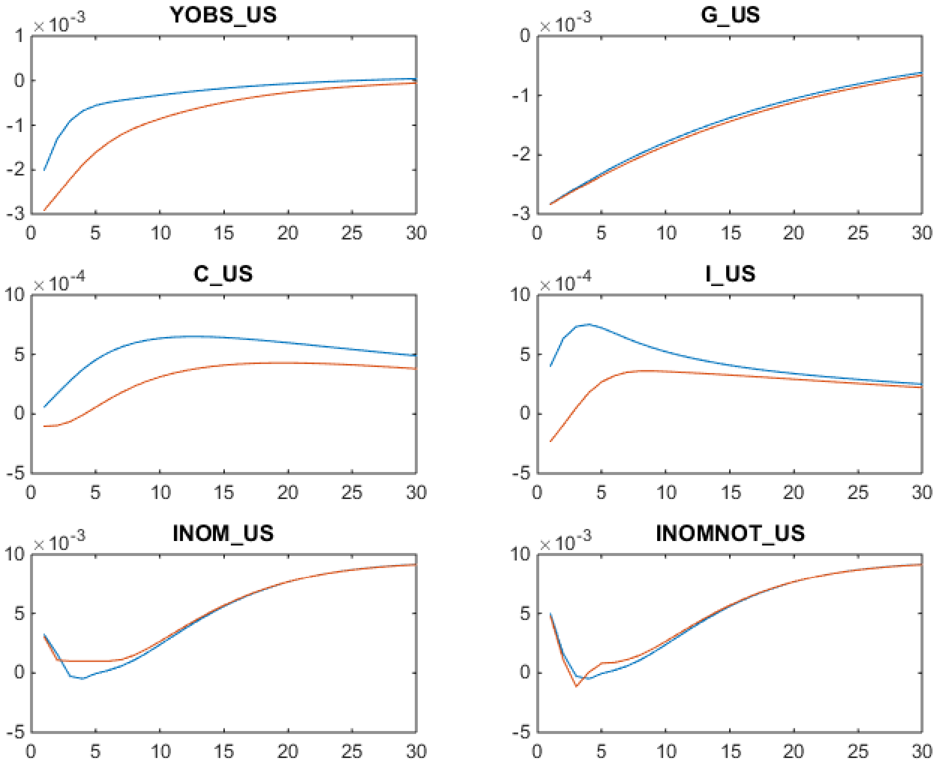
As an example, we perform counterfactual exercises as follows. Using as starting point the smoothed variables in 2008q4, we shut off all fiscal shocks and simulate the model with all other shocks.

We perform another simulation adding a negative government spending shock of -0.25% of quarterly GDP. The difference between the two simulations provides the IRF of a government spending shock under a constrained regime. For both EA and US the multiplier becomes bigger than one and fiscal consolidation generates a *comovement* of consumption and investment with government spending for some periods at the beginning of the simulations.



Negative fiscal shock of 0.25% of GDP in EA in 2009q1, on top of all the other historical shocks. Blue is the linear model, red is the piecewise linear one.

INOM = nominal int. rate; INOMNOT = shadow int. rate



Negative fiscal shock of 0.25% of GDP in US in 2009q1, on top of all the other historical shocks. Blue is the linear model, red is the piecewise linear one.

INOM = nominal int. rate; INOMNOT = shadow int. rate

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