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Liquidity Traps in a World Economy

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This paper studies a New Keynesian model of a two-country world with a zero lower bound (ZLB) constraint for nominal interest rates. A floating exchange rate regime is assumed. The presence of the ZLB generates multiple equilibria. The two countries can experience recurrent liquidity traps induced by the self-fulfilling expectation that future inflation will be low. These "expectations-driven" liquidity traps can be synchronized or unsynchronized across countries. In an expectations-driven liquidity trap, the domestic and international transmission of persistent shocks to productivity and government purchases differs markedly from shock transmission in a "fundamentals-driven" liquidity trap.

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1. Introduction

This paper studies fluctuations of inflation, real activity and the exchange rate in a two-country New Keynesian sticky-prices model. A zero lower bound (ZLB) constraint for nominal interest rates is imposed. When the ZLB binds, i.e. in a "liquidity trap", the central bank cannot stimulate real activity by lowering the policy interest rate (Keynes (1936), Hicks (1937)). The recent experience of persistent low interest rates and low inflation in many advanced economies has led to a resurgence of theoretical research on liquidity traps. Two types of liquidity traps have been discussed in the literature: Firstly, an extensive modeling strand building on Krugman (1998) and Eggertsson and Woodford (2003) considers "fundamentals-driven" liquidity traps that are induced by large shocks to household preferences, or to other fundamentals, which sharply reduce aggregate demand and push the nominal interest rate to the ZLB.[1] Secondly, Benhabib, Schmitt-Grohé and Uribe (2001a,b; 2002a,b) have studied "expectations-driven" liquidity traps, namely liquidity traps that are induced by the self-fulfilling *expectation* that future inflation will be low; Benhabib et al. show that the combination of the ZLB constraint and an "active" Taylor monetary policy interest rate rule gives rise to multiple equilibria, and that expectations-driven liquidity trap can arise *even* when there are no shocks to fundamentals. Fundamentals-driven liquidity traps have been analyzed in both open- and closed economies; [2] by contrast, the literature on expectations-driven liquidity traps has concentrated on closed economies.

The contribution of the present paper is to study expectations-driven sunspot equilibria with occasionally binding ZLB constraints, in open economies; a floating exchange rate regime is assumed.[3] The cause of liquidity traps matters for the dynamics of the world economy. A model with expectations-driven ZLB regimes is better suited for generating persistent liquidity traps than a theory of fundamentals-driven ZLB regimes. A key finding is that expectations-driven ZLB regimes can either be synchronized or unsynchronized across countries: the cross-country correlation of expectations-driven liquidity traps is indeterminate, and unrelated to the

[1] Among many other models with fundamentals-driven liquidity traps, see Christiano et al. (2011), Holden (2016,2019) and Roeger (2015) for detailed references to the related literature.
[2] For analyses of fundamentals-driven liquidity traps in open economies, see, e.g., Jeanne (2009, 2010), Erceg and Lindé (2010), Cook and Devereux (2013, 2016), Fujiwara and Ueda (2013), Gomez et al. (2015), Farhi and Werning (2016), Blanchard et al. (2016), Acharya and Bengui (2018), Corsetti et al. (2018), Fornaro and Romei (2019), Badarau and Sangaré (2019), Balfoussia et al. (2020) and Farhi et al. (2020).
[3] Kollmann (2020) studies expectations-driven liquidity traps, in a model of a currency union, in which liquidity traps are perfectly correlated across countries (all countries face the same policy interest rate). In a floating exchange rate regime (studied here), asynchronous expectations-driven liquidity traps can occur, and exchange rate adjustment plays a key role for domestic and international shock transmission.

correlation of fundamental business cycle shocks. By contrast, the cross-country correlation of fundamentals-driven liquidity traps equals the international correlation of the shocks triggering those traps. I show that the domestic and international transmission of fundamental business cycle shocks (disturbances to productivity, government purchases and household preferences) in an expectations-driven liquidity trap can differ markedly from shock transmission in a fundamentals-driven liquidity trap.

The model variants with expectations-driven liquidity traps studied here postulate that a country's ZLB regime is *solely* driven by agents' self-fulfilling inflation expectations; in those model variants, fundamental shocks are assumed to be sufficiently small, so that fundamental shocks cannot trigger a change in the ZLB regime. This allows a sharp distinction between expectations-driven liquidity traps and fundamentals-driven liquidity traps (that are induced by large fundamental shocks).

Building on Arifovic et al. (2018) and Aruoba et al. (2018), I consider equilibria with expectations-driven liquidity traps in which the decision rule for inflation depends on the ZLB regime and on the natural real interest rate (i.e. the expected real interest rate that would obtain under flexible prices). The natural real interest rate is stationary. Thus, the inflation rate, in an expectations-driven liquidity trap, is likewise stationary.[4] Away from the ZLB, a policy of inflation targeting (implemented via an "active" Taylor rule) ensures that the actual real interest rate tracks the naturel real interest rate. Persistent fundamental shocks only have a muted effect on the natural real interest rate, as the latter is a function of expected growth rates of the fundamental drivers. Away from the ZLB, persistent shocks thus trigger muted responses of inflation. In an expectations-driven liquidity trap, the inflation response to persistent shocks too is muted.

This explains why the transmission of **persistent** fundamental shocks to domestic and foreign real activity and the real exchange rate, in an expectations-driven liquidity trap, is similar to transmission when the economy is away from the ZLB, and to transmission in a flex-prices world. In particular, a persistent positive shock to Home country productivity raises Home output, and it depreciates the Home terms of trade and real exchange rate; a persistent positive shock to Home government purchases raises Home output and appreciates the Home terms of

[4] This explains why an expectations-driven liquidity trap does not exhibit the explosive backward dynamics of inflation and the strong sensitivity to shocks that characterize fundamentals-driven liquidity traps; see below.

trade. For a trade elasticity greater than unity, as assumed in many macro models, the present model with expectations-driven liquidity traps predicts that a persistent rise in Home productivity raises Home net exports and lowers Foreign output, while a persistent rise in Home government purchases lowers Home net exports and raises Foreign output. Domestic and foreign output multipliers of persistent fiscal spending shocks are smaller than unity, in expectations-driven liquidity traps.

A *fundamentals-driven* liquidity trap generates very different responses to persistent shocks. Analyses of fundamentals-driven liquidity traps presented in the literature postulate a baseline liquidity trap scenario in which a large shock to preferences (or other fundamentals) moves the *unconstrained* nominal interest rate into negative territory; the liquidity trap ends when the (mean-reverting) unconstrained nominal rate becomes non-negative again (e.g., Erceg and Lindé (2010), Cochrane (2017)). Inflation during the fundamentals-driven liquidity trap is determined by iterating the Euler and Phillips equations backward, from the trap exit date. The "backward" dynamics of inflation (during the liquidity trap) is explosive. Therefore, small exogenous shocks that are added to the baseline fundamentals-driven liquidity trap scenario can have big effects on inflation, during the liquidity trap. In the model here, a positive Home productivity shock, occurring during a fundamentals-driven liquidity trap, triggers a sizable fall in Home inflation on impact; this sizable drop in inflation *lowers* Home output and consumption and *appreciates* the Home terms of trade and the Home real exchange rate. By contrast, a positive shock to Home government purchases induces a sharp rise in Home inflation, which strongly boosts Home output and *depreciates* the Home real exchange rate. The previous literature on fundamentals-driven liquidity traps has highlighted non-standard (topsy-turvy) output responses to productivity shocks, as well as the large fiscal multipliers in fundamentals-driven liquidity traps (e.g., Eggertsson (2010), Eggertsson and Krugman (2012)). However, the "unorthodox" response of the real exchange rate to productivity and fiscal shocks has apparently not previously been noticed.[5]

I find that *international* spillovers of fundamental business cycle shocks can be much larger and qualitatively different in fundamentals-driven liquidity traps than in expectations-driven liquidity traps. For a trade elasticity greater than unity, model variants with a

[5] Standard macro models predict that, away from the ZLB, a positive shock to a country's productivity depreciates its real exchange rate, while a rise in government purchases appreciates its real exchange rate.

fundamentals-driven liquidity trap predict that a rise in Home productivity lowers Home net exports and raises Foreign output, while a rise in Home government purchases raises Home net exports and lowers Foreign output. These international spillover effects are *opposite* of those predicted in an expectations-driven liquidity trap, with persistent shocks (see above).

Shocks transmission in an expectations-driven liquidity trap is more similar (at least qualitatively) to transmission under a fundamentals-driven liquidity trap, when fundamental shocks are **transitory**. Intuitively, transitory fundamental shocks have a stronger effect on the natural real interest rate than persistent shocks. In a liquidity trap, a transitory shock drives a larger wedge between the actual real interest rate and the natural real interest rate. An "active" Taylor rule implies that, away from the ZLB, the nominal interest rate is cut aggressively in response to a short-lived positive productivity shock, which stabilizes inflation, boosts output and triggers a depreciation in the (nominal and real) exchange rate. In an expectations-driven liquidity trap, the nominal interest rate cannot adjust, which triggers a transitory drop in inflation, a *fall* in domestic consumption and output and an exchange rate appreciation. These responses are qualitatively similar to the responses predicted under a fundamentals-driven liquidity trap.

This paper contributes to a burgeoning literature on business cycle models with expectations-driven liquidity traps, but that literature has assumed closed economies (as mentioned earlier). The paper is related to Mertens and Ravn (2014) who showed, in a closed economy model, that the effect of fiscal shocks differs across expectations-driven and fundamentals-driven liquidity traps (fiscal spending multipliers are smaller in an expectations-driven liquidity trap). Given the recent experience of persistent liquidity traps in several major economies (Euro Area, US, Japan), it is important to study the effect of expectations-driven liquidity traps in models of the *global* economy, for a range of domestic and external shocks. This seems especially relevant as models of fundamentals-driven liquidity traps are assumed in influential policy studies that contribute to the ongoing monetary strategy debates in the US and the Euro Area; see, e.g., Andrade et al. (2019, 2020), Coenen et al. (2020) and Erceg et al. (2020). Other recent studies on expectations-driven liquidity traps include Aruoba et al. (2018), Benigno and Fornaro (2018) and Nakata and Schmidt (2020), who also provide detailed references to the literature. By contrast to the paper here, that literature has not identified the key role of shock *persistence* for the transmission of business cycle shocks, in an expectations-driven liquidity trap.

2. Model of a two-country world

I consider a New Keynesian open economy model with a standard structure of goods, labor and financial markets (e.g., Kollmann (2001, 2002, 2004)). There are two countries, referred to as Home (H) and Foreign (F). Each country has its own currency. The exchange rate is flexible. In each country there are: (i) a central bank that sets the local short-term nominal interest rate; (ii) a government that makes exogenous purchases which are financed using lump-sum taxes; (iii) a representative infinitely-lived household; (iv) monopolistic firms that produce a continuum of differentiated tradable intermediate goods using domestic labor; (v) competitive firms that bundle domestic and imported intermediates into composite non-tradable goods that are used for household and government consumption. Intermediate goods prices are sticky (in producer currency); all other prices are flexible. Each country's household owns the domestic firms, and it supplies labor to those firms (labor is immobile internationally). The labor market is competitive; wages are flexible. For analytical tractability, the model abstracts from physical capital. The Foreign country is a mirror image of the Home country. The following description focuses on the Home country. Analogous conditions describe the Foreign country.

2.1. Home firms

The Home country's household consumes a composite final consumption good $C_{H,t}$ that is produced using the Cobb-Douglas technology $C_{H,t} \equiv (Y^H_{H,t}/\xi)^{\xi} (Y^F_{H,t}/(1-\xi))^{1-\xi}$ where $Y^H_{H,t}$ and $Y^F_{H,t}$ are, respectively, a composite of domestic intermediate goods and a composite of imported intermediates, used by country H. (The superscript on intermediate good quantities denotes the country of origin, while the subscript indicates the destination country.) There is a bias towards using local intermediates, in household consumption: $0.5<\xi<1$. Each country produces a distinct set of intermediates indexed by $s\in[0,1]$. (Intermediate good 's' produced by country H differs from intermediate 's' produced by country F.) The composite intermediate $Y^k_{H,t}$ is given by $Y^k_{H,t} \equiv \{\int_0^1 (y^k_{H,t}(s))^{(\nu-1)/\nu} ds\}^{\nu/(\nu-1)}$ with $\nu>1$, for k=H,F, where $y^k_{H,t}(s)$ is the quantity of the variety s intermediate input produced by country k that is sold to country H, for household consumption.

Home government consumption, denoted $G_{H,t}$, too is a composite of intermediate inputs, but government consumption only uses local intermediates (no imports).[6] Specifically, $G_{H,t} \equiv \{\int_0^1 (g^H_{H,t}(s))^{(\nu-1)/\nu} ds\}^{\nu/(\nu-1)}$, where $g^H_{H,t}(s)$ is the quantity of the Home produced variety s intermediate input that enters Home government consumption.

Let $p_{k,t}(s)$ be the price of intermediate good *s* produced by country *k*, where this price is expressed in country *k* currency. The model assumes producer currency pricing (PCP) for intermediates: intermediate good prices are set (and sticky) in the currency of the country of origin. Home and Foreign intermediate goods markets are integrated. Thus the law of one price holds for intermediates. The price of intermediate *s* produced by country F is $p_{F,t}(s)/S_t$ in the market of country Home, in units if country H currency, where S_t is the nominal exchange rate, defined as the price of a unit of Home currency, in units of Foreign currency. Note that a rise in S_t represents an appreciation of the Home currency.

Cost minimization in Home final good production implies: $y^H_{H,t}(s) = (p_{H,t}(s)/P_{H,t})^{-\nu} Y^H_{H,t}$ and $y^F_{H,t}(s) = ([p_{F,t}(s)/S_t]/P_{H,t})^{-\nu} Y^F_{H,t}$, as well as $Y^H_{H,t} = \xi \cdot CPI_{H,t} \cdot C_{H,t}/P_{H,t}$, $Y^F_{H,t} = (1-\xi) \cdot CPI_{H,t} \cdot C_{H,t}/[P_{F,t}/S_t]$ where $P_{k,t} \equiv \{\int_{s=0}^1 p_{k,t}(s)^{1-\nu} ds\}^{1/(1-\nu)}$ and $CPI_{H,t} \equiv (P_{H,t})^{\xi} (P_{F,t}/S_t)^{1-\xi}$. $P_{k,t}$ is a price index of intermediates produced by country k=H,F, expressed in country k currency. Perfect competition in the final goods market implies that the country H final consumption good price is $CPI_{H,t}$ (its marginal cost). Cost-minimization in Home government consumption requires $g^H_{H,t}(s) = (p_{H,t}(s)/P_{H,t})^{-\nu} G_{H,t}$.

The technology of the firm that produces intermediate good *s* in country H is: $y_{H,t}(s) = \theta_{H,t} L_{H,t}(s)$, where $y_{H,t}(s)$ and $L_{H,t}(s)$ are the firm's output and labor input at date *t*, while $\theta_{H,t} > 0$ is exogenous productivity in country H (all intermediate good producers located in a given country have identical productivity). The firm's good is sold domestically and exported: $y_{H,t}(s) = y^H_{H,t}(s) + y^H_{F,t}(s) + g^H_{H,t}(s)$.

[6] Empirically, the import content of government spending is much lower than that of private consumption (e.g., Bussière et al. (2013). The main results below do not depend on assuming that the government consumption basket differs from the household consumption basket.

Intermediate good producers face quadratic costs to adjusting their prices. The real profit, in units of Home consumption, of the firm that produces Home intermediate good *s* is:

$$\pi_{H,t}(s) \equiv (p_{H,t}(s) - W_{H,t}/\theta_{H,t})y_{H,t}(s)/CPI_{H,t} - \tfrac{1}{2}\psi \cdot ([p_{H,t}(s) - \Pi \cdot p_{H,t-1}(s)]/P_{H,t-1})^2, \quad \psi>0$$

where $W_{H,t}$ is the nominal wage rate in country H. The last term in the profit equation is the real price adjustment cost, where $\Pi > 1$ is the central bank's gross inflation target (see below). The firm sets $p^H_{H,t}(s)$ to maximize the present value of profits $E_t \sum_{\tau=0}^{\infty} \rho^H_{t,t+\tau} \pi_{H,t+\tau}(s)$, where $\rho^H_{t,t+\tau}$ is the Home household's intertemporal marginal rate of substitution in consumption between periods t and $t+\tau$. All Home intermediate good firms face identical decision problems, and they set identical prices: $p_{H,t}(s)=P_{H,t}$ $\forall$ $s\in[0,1]$. The labor input and output are also equated across all Home intermediate good firms.

The Home terms of trade and the real exchange rate (CPI-based) are $q_t \equiv S_t P_{H,t}/P_{F,t}$ and $RER_t \equiv S_t CPI_{H,t}/CPI_{F,t}$, respectively. Note that $RER_t = (q_t)^{2\xi-1}$. Due to household consumption home bias (2ξ-1>0), the real exchange rate is an increasing function of the terms of trade. The real price of the domestic intermediate good, in units of final consumption, is likewise an increasing function of the terms of trade:

$$P_{H,t}/CPI_{H,t} = (q_t)^{1-\xi}. \tag{1}$$

2.2. Household preferences and labor supply

The intertemporal preferences of the representative Home household are described by $E_0 \sum_{t=0}^{\infty} \beta^t \Psi_{H,t} U(C_{H,t}, L_{H,t})$ where $C_{H,t}$ and $L_{H,t}$ are final consumption and aggregate hours worked, respectively. $0<\beta<1$ is the household's steady state subjective discount factor and $U(C_{H,t}, L_{H,t}) = \ln(C_{H,t}) - \frac{1}{1+1/\eta}(L_{H,t})^{1+1/\eta}$ is the agent's period utility function, where $\eta>0$ is the Frisch labor supply elasticity. $\Psi_{H,t} > 0$ is a stationary exogenous preference shock that alters the household's rate of time preference. The household equates the marginal rate of substitution between leisure and consumption to the real wage rate, which implies

$$(1/C_{H,t})(W_{H,t}/CPI_{H,t}) = (L_{H,t})^{1/\eta}. \tag{2}$$

2.3. Financial markets

The model assumes complete international financial markets, and so consumption risk is efficiently shared across countries. In equilibrium, the ratio of Home to Foreign households' marginal utilities of consumption is, thus, proportional to the Home real exchange rate (Kollmann (1991, 1995); Backus and Smith (1993)): $\{\Psi_{H,t}/C_{H,t}\}/\{\Psi_{F,t}/C_{F,t}\}=\Lambda RER_t$, where Λ is a date- and state-invariant term that reflects the (relative) initial wealth of the two countries. I assume that the two countries have the same initial wealth, i.e. $\Lambda=1$. Thus:

$$C_{H,t}/C_{F,t}=(\Psi_{H,t}/\Psi_{F,t})/RER_t. \tag{3}$$

There is also a market for one-period riskless nominal bonds (in zero net supply) that are denominated in Home and in Foreign currency, respectively. Let $i_{k,t+1}$ denote the nominal interest rate on the bond denominated in country k currency, between periods t and t+1. The Home household's Euler equation for the Home currency bond is:

$$(1+i_{H,t+1})E_t\beta(\Psi_{H,t+1}/\Psi_{H,t})(C_{H,t}/C_{H,t+1})/\Pi^{CPI}_{H,t+1}=1, \tag{4}$$

where $\Pi^{CPI}_{H,t+1}\equiv CPI_{H,t+1}/CPI_{H,t}$ is the Home gross CPI inflation rate between periods t and t+1.

2.4. Monetary policy

The Home country's central bank sets the interest rate $i_{H,t+1}$ according to a feedback rule that targets $\Pi_{H,t}\equiv P_{H,t}/P_{H,t-1}$, the gross inflation rate of the Home *producer price index (PPI)*, subject to the zero lower bound (ZLB) constraint $i_{H,t+1}\geq 0$. Specifically, the monetary policy rule is

$$1+i_{H,t+1}=Max\{1,\Pi/\beta+(\gamma_\pi/\beta)\cdot(\Pi_{H,t}-\Pi)\}\,,\gamma_\pi>1 \tag{5}$$

where $\Pi>1$ is the central bank's gross inflation target. Π/β is the gross nominal interest rate that obtains when the inflation rate equals the central bank's inflation target. γ_π is a parameter that captures the central bank's policy response to inflation. The "Taylor principle" ($\gamma_\pi>1$) is assumed to hold ("active" monetary policy), when the ZLB constraint is slack: then, a rise in inflation by 1 percentage point (ppt) triggers a rise of the policy rate by more than 1 ppt.

2.5. Market clearing

Market clearing in the country k=H,F labor market requires $L_{k,t}=\int_{s=0}^{1} L_{k,t}(s)\,ds$. Real GDP ($Y_{k,t}$) equals aggregate intermediate good output, $Y_{k,t}=\theta_{k,t}L_{k,t}$. Markets for individual intermediates clear as intermediate good firms meet all demand at posted prices. This implies $Y_{k,t}=Y_{H,t}^{k}+Y_{F,t}^{k}+G_{k,t}$ i.e. aggregate intermediate good output equals the sum of aggregate domestic and foreign intermediate good demand. Using the intermediate good demand functions, this condition can be expressed as $Y_{H,t}=\xi CPI_{H,t}C_{H,t}/P_{H,t}+(1-\xi)CPI_{F,t}C_{F,t}/[S_tP_{H,t}]+G_{H,t}$ and $Y_{F,t}=(1-\xi)CPI_{H,t}C_{H,t}/[P_{F,t}/S_t]+\xi CPI_{F,t}C_{F,t}/P_{F,t}+G_{F,t}$.

2.6. Solving the model

Following much of the previous literature on macro models with a ZLB constraint (see Holden (2016, 2019) for detailed references), I linearize all equations, **with the exception of the monetary policy rule (5)**. This allows to capture the macroeconomic effects of the occasionally binding ZLB constraint, while keeping analytical tractability.

I take a linear approximation around a symmetric steady state in which (in both countries) the gross inflation rate equals the inflation target Π; the corresponding steady state gross interest rate is $1+i=\Pi/\beta$. Let $\widehat{x}_t\equiv(x_t-x)/x$ denote the relative deviation of a variable x_t from its steady state value $x\neq 0$ (variables without time subscript denote steady state values). To simplify the analytical expressions, I assume that government purchases are zero, in steady state. [7] I define $\widehat{G_{k,t}}\equiv G_{k,t}/Y_k$ as the ratio of government purchases to steady state GDP in country k=H,F.

Linearization of the risk-sharing condition (3) and of the (intermediate) goods market clearing conditions gives:

$$\widehat{C_{H,t}}-\widehat{C_{F,t}}=-(2\xi-1)\widehat{q}_t+\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}}\,, \tag{6}$$

$$\widehat{Y_{H,t}}=\xi\widehat{C_{H,t}}+(1-\xi)\widehat{C_{F,t}}-2\xi(1-\xi)\widehat{q}_t+\widehat{G_{H,t}},\quad \widehat{Y_{F,t}}=(1-\xi)\widehat{C_{H,t}}+\xi\widehat{C_{F,t}}+2\xi(1-\xi)\widehat{q}_t+\widehat{G_{F,t}}\,. \tag{7}$$

The linearized bond Euler equation (4) of country k=H,F is:

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[7]The analysis below will allow for both positive and negative shocks to government purchases. An interpretation of negative government purchases is that government occasionally has an autonomous supply of resources that it distributes to the private sector.

$$\widehat{1+i_{k,t+1}}=E_t\{\widehat{\Pi^{CPI}_{k,t+1}}+\widehat{C_{k,t+1}}-\widehat{C_{k,t}}+\widehat{\Psi_{k,t}}-\widehat{\Psi_{k,t+1}}\}. \tag{8}$$

Linearizing the first-order condition of the intermediate good firms' decision problem in country k=H,F gives a standard 'forward-looking' Phillips equation:

$$\widehat{\Pi_{k,t}}=\kappa_w\cdot\widehat{mc_{k,t}}+\beta E_t\widehat{\Pi_{k,t+1}}, \tag{9}$$

where $\Pi_{k,t}\equiv P_{k,t}/P_{k,t-1}$, while $mc_{k,t}=(W_{k,t}/\theta_{k,t})/P_{k,t}$ is real marginal cost, deflated by the domestic producer price index, in country k's intermediate good sector (e.g., Kollmann (2002)). $\kappa_w>0$ is a coefficient that is a decreasing function of the price adjustment-cost parameter ψ. Using the nominal wage rate implied by the Home household's labor supply equation (2) (and the analogous Foreign equation) allows to express Home and Foreign real marginal costs as:

$$\widehat{mc_{H,t}}=\widehat{C_{H,t}}+\tfrac{1}{\eta}\widehat{Y_{H,t}}-(1+\tfrac{1}{\eta})\widehat{\theta_{H,t}}-(1-\xi)\widehat{q_t} \text{ and } \widehat{mc_{F,t}}=\widehat{C_{F,t}}+\tfrac{1}{\eta}\widehat{Y_{F,t}}-(1+\tfrac{1}{\eta})\widehat{\theta_{F,t}}+(1-\xi)\widehat{q_t}. \tag{10}$$

Expressing the monetary policy interest rate rule (5) using 'hatted' variables gives

$$\widehat{(1+i_{k,t+1})}=Max\{-(\Pi-\beta)/\Pi,\gamma_\pi\cdot\widehat{\Pi_{k,t}}\}. \tag{11}$$

Note that the interest rate $\widehat{(1+i_{k,t+1})}$ is a *non-linear* function of inflation. The ZLB constraint binds when $\gamma_\pi\widehat{\Pi_{k,t}}\le-(\Pi-\beta)/\Pi$.

Using the risk sharing condition (6), the market clearing conditions (7) can be written as:

$$\widehat{Y_{H,t}}=Z_{H,t}-(1-\xi)(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}})+\widehat{G_{H,t}} \quad\text{and}\quad \widehat{Y_{F,t}}=Z_{F,t}+(1-\xi)(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}})+\widehat{G_{F,t}}, \tag{12}$$

where $Z_{H,t}\equiv\widehat{C_{H,t}}-(1-\xi)\widehat{q_t}$ and $Z_{F,t}\equiv\widehat{C_{F,t}}+(1-\xi)\widehat{q_t}$. Substitution of (12) into (10) allows to express real marginal cost in country k as a function of $Z_{k,t}$:

$$\widehat{mc_{k,t}}=(1+\tfrac{1}{\eta})Z_{k,t}-(1+\tfrac{1}{\eta})\widehat{\theta_{k,t}}+\tfrac{1}{\eta}\widehat{G_{k,t}}-\tfrac{1}{\eta}(1-\xi)(\widehat{\Psi_{k,t}}-\widehat{\Psi_{l,t}}), \text{ for } k,l\in\{\text{H,F}\},\ k\neq l. \tag{13}$$

Using (1), the growth factor of country k nominal consumption spending can be expressed as

$$\widehat{\Pi^{CPI}_{k,t+1}}+\widehat{C_{k,t+1}}-\widehat{C_{k,t}}=\widehat{\Pi_{k,t+1}}+Z_{k,t+1}-Z_{k,t}. \tag{14}$$

Using (14), the Euler equation (8) of country k=H,F can be written in terms of PPI inflation and the expected future change of Z:

$$\widehat{1+i_{k,t+1}}=E_t\{\widehat{\Pi_{k,t+1}}+Z_{k,t+1}-Z_{k,t}+\widehat{\Psi_{k,t}}-\widehat{\Psi_{k,t+1}}\}. \tag{15}$$

Next, combine the Euler equation (15) and the interest rate rule (11), and substitute out Z_k using the formula for real marginal cost (13) and the Phillips equation (9). This gives the following non-linear equation that governs the dynamics of PPI inflation in country *k*:

$$Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi_{k,t}}\} + \tfrac{1}{\kappa}\widehat{\Pi_{k,t}} = (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi_{k,t+1}} - \tfrac{\beta}{\kappa}E_t\widehat{\Pi_{k,t+2}} + \widehat{r_{k,t}}\,, \tag{16}$$

with $\kappa \equiv \frac{1+\eta}{\eta}\kappa_w$ and

$$\widehat{r_{k,t}} \equiv E_t\widehat{\theta_{k,t+1}} - \widehat{\theta_{k,t}} - \tfrac{1}{1+\eta}E_t(\widehat{G_{k,t+1}} - \widehat{G_{k,t}}) - [\xi + \tfrac{\eta}{1+\eta}(1-\xi)]E_t(\widehat{\Psi_{k,t+1}} - \widehat{\Psi_{k,t}}) - \tfrac{1}{1+\eta}(1-\xi)E_t(\widehat{\Psi_{l,t+1}} - \widehat{\Psi_{l,t}}),$$

for $k,l \in \{\text{H,F}\}$, $k \neq l$.

I will call (16) the "Euler-Phillips" equation of country k. $\widehat{r_{k,t}}$ is a function of exogenous variables. In a flex-prices world κ=∞ holds, and the Euler-Phillips equation (16) becomes $\widehat{1+i_{k,t+1}} - E_t\widehat{\Pi_{k,t+1}} = \widehat{r_{k,t}}$. Thus, $\widehat{r_{k,t}}$ is the country k expected gross real interest rate (expressed as a relative deviation from the steady state gross real rate), defined in units of country k output, that would obtain in a flex-prices world. I refer to $\widehat{r_{k,t}}$ as country k's natural real interest rate. $\widehat{r_{k,t}}$ only depends on fundamental exogenous forcing variables.

To solve the model, we have to find processes for Home and Foreign inflation that solve the Euler-Phillips equation (16) for k=H,F. Once such processes have been determined, GDP (aggregate output), consumption, the terms of trade and net exports can be determined using the Phillips equation (9) and the static model equations (see Appendix B).

Note that, in the baseline model considered here, the two countries' Euler-Phillips equations are uncoupled, in the sense that the country k Euler-Phillips equation involves domestic inflation, but not foreign inflation. The natural real interest rate is a function of domestic productivity and government purchases, but not of foreign productivity and government purchases. This helps to understand why, in equilibrium, productivity and government purchases shocks have zero spillovers to foreign output and inflation, as shown by the simulations below (however, there are non-zero spillovers to foreign consumption, due to international risk sharing). Net exports too are unaffected by productivity and government purchases shocks, in the baseline model.

The zero international output spillovers of productivity and government purchases shocks reflect the household preferences of the Cole and Obstfeld (1991) type assumed here, i.e. the combination of a unitary intertemporal consumption substitution elasticity and a unitary trade elasticity (substitution elasticity between domestic and imported intermediates); see further discussion below. I use this specification as it greatly simplifies the analysis and the presentation. In a sensitivity analysis below, I consider a model variant with a non-unitary trade elasticity; that model variant generates non-zero cross-country spillovers (see Sect. 5).

The subsequent discussion assumes that productivity, government purchases and the preference shifter Ψ follow stationary univariate AR(1) processes with a common autocorrelation $0 \leq \rho < 1$: $\widehat{\theta_{k,t+1}} = \rho \widehat{\theta_{k,t}} + \varepsilon^{\theta}_{k,t+1}$, $\widehat{G_{k,t+1}} = \rho \widehat{G_{k,t}} + \varepsilon^{G}_{k,t+1}$, $\widehat{\Psi_{k,t+1}} = \rho \widehat{\Psi_{k,t}} + \varepsilon^{\Psi}_{k,t+1}$ for k=H,F where $\varepsilon^{\theta}_{k,t+1}, \varepsilon^{G}_{k,t+1}, \varepsilon^{\Psi}_{k,t+1}$ are exogenous mean-zero innovations. This implies that natural real interest rates too follow AR(1) processes with autocorrelation ρ. Note that

$$\widehat{r_{k,t}} \equiv (1-\rho)\{-\widehat{\theta_{k,t}} + \tfrac{1}{1+\eta}\widehat{G_{k,t}} + [\xi + \tfrac{\eta}{1+\eta}(1-\xi)]\widehat{\Psi_{k,t}} + \tfrac{1}{1+\eta}(1-\xi)\widehat{\Psi_{l,t}}\} \quad \text{for } k,l \in \{\text{H,F}\},\ k \neq l. \tag{17}$$

The country k natural real interest rate is a decreasing function of domestic productivity and an increasing function of domestic government purchases and of domestic and foreign preference shocks. Because of the assumed mean reversion of productivity, a positive productivity shock reduces the expected *future* growth rate of productivity; in a flex-prices economy, a positive productivity shock increases consumption on impact; future consumption rises less than current consumption, i.e. the expected growth rate of consumption falls, and hence the real natural interest rate drops. A similar logic explains why positive fiscal spending and preference shocks raise the natural real interest rate.

2.7. Flex-prices world

In the sticky-prices world, Home and Foreign monetary policies that fully stabilize the domestic PPI inflation rate at the central bank's inflation target, so that $\widehat{\Pi_{k,t}} = 0 \ \forall t$, would ensure that output, consumption, net exports and the terms of trade equal the flex-prices allocation.[8] This implies that, if inflation responses to exogenous shocks are sufficiently muted in a sticky-prices world, the transmission of those shocks to real activity, net exports and the terms of trade

[8] Under flexible prices, real marginal cost is constant. The flex-prices allocation can be solved for using the risk-sharing and market clearing conditions (6),(7), and mark-up equations (10), with $\widehat{mc_{H,t}} = \widehat{mc_{K,t}} = 0$. Under sticky prices, a monetary policy that fully stabilizes PPI inflation (at the inflation target Π) stabilizes real marginal cost (see (9)), and thus it reproduces the flex-prices allocation. When there is a ZLB constraint, the central bank cannot guarantee full PPI inflation stabilization, because of the existing of multiple equilibria (see below).

resembles transmission under flexible prices. Therefore, a flex-prices (Real Business Cycle) model provides a useful benchmark for understanding the dynamics of real variables in the sticky-prices world. The solution of the linearized flex-prices model is:

$$\widehat{Y_{k,t}}=\widehat{\theta_{k,t}}+\tfrac{\eta}{1+\eta}\widehat{G_{k,t}}-\tfrac{(1-\xi)\eta}{1+\eta}\cdot(\widehat{\Psi_{k,t}}-\widehat{\Psi_{l,t}}), \quad \text{for } k,l\in\{\text{H,F}\},\ k\neq l;$$

$$\widehat{C_{k,t}}=\xi\widehat{\theta_{k,t}}+(1-\xi)\widehat{\theta_{k,t}}-\tfrac{1}{1+\eta}[\xi\widehat{G_{k,t}}+(1-\xi)\widehat{G_{l,t}}]+\tfrac{(1-\xi)(2\xi+\eta)}{1+\eta}\cdot(\widehat{\Psi_{k,t}}-\widehat{\Psi_{l,t}}) \quad \text{for } k,l\in\{\text{H,F}\},\ k\neq l;$$

$$\widehat{q_t}=-(\widehat{\theta_{H,t}}-\widehat{\theta_{F,t}})+\tfrac{1}{1+\eta}\cdot(\widehat{G_{H,t}}-\widehat{G_{F,t}})+\tfrac{2\xi-1+\eta}{1+\eta}\cdot(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}});$$

$$NX_{k,t}=-(1-\xi)\cdot(\widehat{\Psi_{k,t}}-\widehat{\Psi_{l,t}}) \text{ for } k,l\in\{\text{H,F}\},\ k\neq l,$$

where $NX_{k,t}$ denotes country k net exports (normalized by GDP). [9]

Flex-prices output is an increasing function of domestic productivity and domestic government purchases, but output does not depend on foreign productivity and foreign government purchases. With flexible prices, a positive Home productivity shock increases Home output, and it raises the relative price of the Foreign-produced good; thus, the shock has opposing income and substitution effects on the demand for Foreign output. The improvement in the Foreign terms of trade triggered by the shock raises the Foreign real consumption wage, which has opposing wealth and substitution effects on Foreign labor supply. Under the Cole-Obstfeld preference specification, these opposing effects cancel out, and Foreign output does not respond to the Home productivity shock. Note that productivity and government purchases shocks do not affect net exports. Under flexible prices, the Home terms of trade are a decreasing function of relative (Home vs. Foreign) productivity and an increasing function of relative government purchases. A positive country k preference shock raises k's consumption, and lowers k's output (as the rise in consumption triggers a fall in labor supply). The terms of trade are an increasing function of a country's relative preference shock, under flexible prices.

2.8. Model calibration

The model simulations discussed below assume that one period in the model represents one quarter in calendar time. I set β=0.9975, which implies a 1% per annum steady state riskless real

[9] $NX_{k,t}\equiv\{P_{k,t}Y_{k,t}-CPI_{k,t}C_{k,t}-P_{k,t}G_{k,t}\}/\{P_{k,t}Y_{k,t}\}$. Up to a linear approximation, $NX_{H,t}=\widehat{Y_{H,t}}-\widehat{C_{H,t}}-\widehat{G_{H,t}}+(1-\xi)\widehat{q_t}$ and $NX_{F,t}=\widehat{Y_{F,t}}-\widehat{C_{F,t}}-\widehat{G_{F,t}}-(1-\xi)\widehat{q_t}$.

interest rate. The Frisch labor supply elasticity is set at unity, η=1, a conventional value in macro models. The local content of private consumption spending is set at ξ=0.87. [10] The Central Bank's quarterly gross inflation target is set at Π=1.005, in line with a 2% annual inflation target. The inflation coefficient of the interest rate rule (5) is set at the conventional value γ_π=1.5. The slope coefficient κ_w of the Phillips equation (9) is set at a value such that the observationally equivalent Phillips curve implied by Calvo (1983) staggered price setting entails an average duration between price changes of 4 quarters. This mean duration is consistent with empirical evidence on price setting in the Euro Area and the US (see Kollmann (2001), Alvarez et al. (2006), Giovannini et al. (2019)).[11] *The preceding parameters are used in all simulations below.*

For comparison purposes with the simulations of the sticky-prices model, I note that, in the flex-prices model (with baseline parameters), the decision rules for Home output, consumption, net exports and the terms of trade are:

$$\widehat{Y_{H,t}}=\widehat{\theta_{H,t}}+0.50\cdot\widehat{G_{H,t}}-0.06\cdot(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}}),\ \ \widehat{C_{H,t}}=0.87\cdot\widehat{\theta_{H,t}}+0.13\cdot\widehat{\theta_{F,t}}-0.44\cdot\widehat{G_{H,t}}-0.06\cdot\widehat{G_{F,t}}+0.18\cdot(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}}),$$

$$NX_{H,t}=-0.13\cdot(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}}),\ \ \widehat{q_t}=-(\widehat{\theta_{H,t}}-\widehat{\theta_{F,t}})+0.50\cdot(\widehat{G_{H,t}}-\widehat{G_{F,t}})+0.87\cdot(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}})\,. \qquad (18)$$

3. Expectations-driven ZLB regimes

In this Section, I construct equilibria with expectations-driven ZLB regimes. I begin by characterizing steady state equilibria and then consider equilibria with stochastic fundamental shocks and time-varying ZLB regimes. Building on Arifovic et al. (2018) and Aruoba et al. (2018) (who analyzed expectations-driven liquidity traps in closed-economy models), I consider equilibria in which the ZLB regime follows a Markov chain, and in which PPI inflation in country k=H,F is solely a function of the country's ZLB regime and of the natural real interest rate. For given transition probabilities between the expectations-driven ZLB regimes, the regime-specific inflation decisions rules studied here are unique.

[10] This value of ξ matches the fact that, empirically, the US trade share was 13% in the period 1990-2019.

[11] Under Calvo price setting, the slope of the Phillips curve (9) is $\kappa_w=(1-D)(1-\beta D)/D$, where 1-*D* is the probability that an individual firm can change its price in a given period, so that the average duration between price changes is 1/(1-*D*). I set *D*=0.75 (average stickiness of 4 periods), which implies κ_w=0.08395.

The present analysis focuses entirely on Benhabib-Schmitt-Grohé-Uribe type expectations-driven indeterminacy of the ZLB regime. In the multiple equilibria studied here, the ZLB regime can be interpreted as determined by an "extrinsic" sunspot variable; inflation *within* each ZLB regime is assumed to be solely a function of fundamental exogenous variables (via the natural real interest rate). It should be noted that there also exist equilibria in which inflation, during a liquidity trap, depends on additional sunspots (i.e. on sunspots other than the sunspot variable that determines the ZLB regime).[12] This reflects the local indeterminacy induced by the violation of the Taylor principle, in a liquidity trap. Analysis of this additional dimension of indeterminacy, in the open economy model here, is left for future research. See Lubik and Schorfheide (2003, 2004), Cho and Moreno (2011) and Farmer et al. (2015) for closed economy analyses (without the ZLB regime shifts studied in the present paper) of multiple sunspot equilibria induced by violations of the Taylor principle. [13]

3.1. Steady state equilibria

The model has multiple bounded solutions. To see this in the simplest possible way, consider first a world without shocks to the natural real interest rates: $\widehat{r_{k,t}}=0\ \ \forall t$. The steady-state Euler-Phillips equation is (from (16)): $Max\{-(\Pi-\beta)/\Pi, \gamma_{\pi}\cdot\widehat{\Pi_k}\}=\widehat{\Pi_k}$ for k=H,F. Given our assumption that $\gamma_{\pi}>1$, this equation is solved by two steady state (constant) inflation rates: $\widehat{\Pi_k}=0$ and $\widehat{\Pi_k}=-(\Pi-\beta)/\Pi$. The ZLB binds in the latter steady state. In the steady state liquidity trap, agents expect that future inflation will be low in all periods, which implies that current inflation is low, thus causing the ZLB constraint to bind; in other terms, the liquidity trap is "expectations-driven". The multiplicity of equilibria here is in line with Benhabib et al. (2001a,b) who showed (in a simpler model) that the combination of the ZLB and an "active" Taylor rule produced two steady states and that the ZLB binds in one of these steady states. Note that a steady state

[12] The online Appendix of Aruoba et al. (2018) constructs an equilibrium (closed economy) with two independent sunspots: one sunspot determines the ZLB regime; the other sunspot affects inflation *during* the liquidity trap.

[13] In the context of New Keynesian closed economy models with exogenous switches between monetary policy regimes, Davig and Leeper (2007, 2010), Farmer et al. (2010), Cho (2016), Barthélemy and Marx (2019) and Neusser (2019) construct multiple equilibria in which inflation depends on sunspots, during regimes with Taylor-principle violations.

liquidity trap can arise in country H, irrespective of whether there is a liquidity trap in country Foreign, and vice versa.

3.2. Equilibria with shocks to natural real interest rates

I now construct multiple equilibria for a world with time-varying natural real interest rates. The model variants with expectations-driven ZLB regimes considered here postulate that a country's current ZLB regime is *solely* driven by agents' self-fulfilling beliefs about future ZLB regimes. In those model variants, it is postulated that fundamental shocks are sufficiently small, so that fundamental shocks cannot trigger a change in the ZLB regime. This allows a sharp distinction between expectations-driven liquidity traps and fundamentals-driven liquidity traps (that are induced by large fundamental shocks); see below.

Following Arifovic et al. (2018) and Aruoba et al. (2018), I consider equilibria in which the decision rule for PPI inflation depends solely on the ZLB regime and on the natural real interest rate. Because, in the baseline model, the two countries' Euler-Philipps equations are uncoupled, the equilibrium decision rule for country k PPI inflation only depends on that country's ZLB regime, but not on the ZLB regime of the other country. Denote country k inflation when the ZLB constraint binds by $\widehat{\Pi_{k,t}^B}$, and by $\widehat{\Pi_{k,t}^S}$ when the ZLB constraint is slack. The inflation decision rules in these two regimes are of the form

$$\widehat{\Pi_{k,t}^B} = \mu^B + \lambda^B \widehat{r_{k,t}} \tag{19}$$

$$\text{and} \quad \widehat{\Pi_{k,t}^S} = \mu^S + \lambda^S \widehat{r_{k,t}}\ , \tag{20}$$

$$\text{with} \quad \gamma_\pi \widehat{\Pi_{k,t}^B} \leq -(\Pi - \beta)/\Pi < \gamma_\pi \widehat{\Pi_{k,t}^S}. \tag{21}$$

For given transition probabilities between ZLB regimes (see below), the coefficients of the decision rules $\mu^B, \lambda^B, \mu^S, \lambda^S$ can be found using the method of undetermined coefficients, by substituting (19) and (20) into the Euler-Phillips equation (16).

I will first consider equilibria in which each country is in a permanent ZLB regime, as closed-form model solutions can easily be derived for that case. I then consider equilibria with time-varying ZLB regimes.

3.2.1. Home country in permanent expectations-driven liquidity trap

This Section studies an equilibrium with time-varying natural real interest rates in which agents rationally believe that the Home economy will forever be in a liquidity trap, so that $\widehat{\Pi_{H,t}}=\widehat{\Pi^B_{H,t}}$ $\forall t$ (see (19)). Then, the Home Euler-Phillips equation (16) becomes:

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\widehat{\Pi^B_{H,t}} + (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi^B_{H,t+1}} - \tfrac{\beta}{\kappa}E_t\widehat{\Pi^B_{H,t+2}} + \widehat{r_{H,t}}. \qquad (22)$$

Substitution of the decision rule (19) into (22) gives:

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\{\mu^B+\lambda^B\widehat{r_{H,t}}\} + (1+\tfrac{1+\beta}{\kappa})\{\mu^B+\lambda^B\rho\widehat{r_{H,t}}\} - \tfrac{\beta}{\kappa}\{\mu^B+\lambda^B\rho^2\widehat{r_{H,t}}\} + \widehat{r_{H,t}}, \qquad (23)$$

where I use the fact that (19) implies $\mathrm{E}_t\widehat{\Pi^B_{H,t+s}}=\mu^B+\lambda^B\rho^s\widehat{r_{H,t}}$ for s≥0.

(23) holds for arbitrary values of $\widehat{r_{H,t}}$ iff $\mu^B=-(\Pi-\beta)/\Pi$ and $\{-\tfrac{1}{\kappa}+(1+\tfrac{1+\beta}{\kappa})\rho-\tfrac{\beta}{\kappa}\rho^2\}\lambda^B+1=0$. Thus, the slope of the decision rule in a permanent liquidity trap is: $\lambda^B=-1/\{-\tfrac{1}{\kappa}+(1+\tfrac{1+\beta}{\kappa})\rho-\tfrac{\beta}{\kappa}\rho^2\}$. This can be written as $\lambda^B=-(\kappa/\beta)/\Gamma(\rho)$, where $\Gamma(\rho)\equiv-\rho^2+(1+\tfrac{1+\beta}{\kappa})\rho-\tfrac{1}{\beta}$. Note that $\Gamma(0)=-\tfrac{1}{\beta}<0$ and $\Gamma(1)=\tfrac{\kappa}{\beta}>0$; furthermore $\Gamma'(\rho)>0$ for $0\leq\rho\leq1$. Therefore, $\Gamma(\rho)>0$ holds for $0<\Xi<\rho\leq1$, where Ξ is the root of the polynomial $\Gamma(\Xi)=0$. For the values of β,κ assumed in the model calibration (see above), we have Ξ=0.67. Empirical estimates of the quarterly autocorrelation of productivity, government purchases (and other macroeconomic shocks) are typically in the range between 0.95 and 1, and thus clearly larger than Ξ.[14] This implies that $\Gamma(\rho)>0$ holds for an autocorrelation ρ in the empirically relevant range. For plausible ρ, we thus have $\lambda^B<0$, which implies that a rise in the natural interest rate lowers the inflation rate, in a permanent liquidity trap, so that inflation is *increasing* in productivity, and *decreasing* in government purchases and the preference shifter (as the natural real interest rate is a decreasing function of productivity, and an increasing function of government purchases and of the preference shifter Ψ ; see (17)).

For intuition, note that a persistent rise in the natural real interest rate induces a rise in the expected future real interest rate. In a permanent expectations-driven liquidity trap, the nominal

[14] King and Rebelo (1999) report an empirical estimate of ρ=0.979 for quarterly US total factor productivity. For the Euro Area (EA) and the US, the autocorrelations of linearly detrended quarterly real government purchases was 0.98, in 1999q1-2017q4; the autocorrelations of EA and US government purchases/GDP ratio were 0.96 and 0.98, respectively.

interest rate is stuck at zero, and the rise in the real interest rate is brought about by a fall in the inflation rate. This can be seen most easily when ρ is very close to (but below) unity. Then $\widehat{\Pi^B_{H,t}} \approx E_t \widehat{\Pi^B_{H,t+1}} \approx E_t \widehat{\Pi^B_{H,t+2}}$, and (22) gives $\widehat{\Pi^B_{H,t}} \approx -(\Pi-\beta)/\Pi - \widehat{r_{H,t}}$, so that a positive shock to the natural real rate triggers (approximately) a one-to-one negative response of the current and expected future inflation rate.

By contrast, when the natural real interest rate is less persistent, $\rho<\Xi$, then a positive shock to the Home natural real interest rate *raises* the Home inflation rate, in a permanent expectations-driven liquidity trap, and hence a positive productivity shock *lowers* domestic inflation. This can be seen most easily when $\rho=0$. A one-time Home productivity increase at date *t* lowers the natural real interest rate at *t*, but it has zero effect on the natural real interest rate at *t*+1; thus, the shock has zero effect on Home inflation at *t*+1, which implies that the shock also has zero effect on Home output and consumption at *t*+1. The Home Euler equation between t and *t*+1 shows that, hence, consumption at date *t* does not respond to the shock, in a liquidity trap (as then the nominal interest rate cannot adjust to the shock). The Home inflation rate at *t* falls to offset the stimulative effect of the one-time productivity increase on Home output, and thereby ensure that Home consumption (and output) remain unchanged at *t*. (When the serial correlation of productivity is strictly positive but smaller than Ξ, then it remains true that a positive Home productivity shock lowers Home inflation, in an expectations-driven liquidity trap, but the shock reduces Home output and consumption; see simulations for the case ρ=0.5 in Sect. 5.)

Unless stated otherwise, the following simulations assume $\rho=0.95$, so that $\lambda^B<0$. Autocorrelations equal, or close to, 0.95 are widely assumed in macroeconomic models.

Inflation in a permanent liquidity trap has to satisfy the restriction $\gamma_\pi \widehat{\Pi^B_{H,t}} \leq -\{(\Pi-\beta)/\Pi\}$ (see (21)), i.e. inflation has to remain sufficiently low to ensure that the ZLB constraint binds. When $\lambda^B<0$ holds, this requires $\widehat{r_{H,t}} \geq (1/\lambda^B)\{(\gamma_\pi-1)/\gamma_\pi\}(\Pi-\beta)/\Pi$, where the right-hand side is negative; thus the natural rate cannot drop too much (to prevent a change in the ZLB regime).

For $\rho=0.95$, the decision rule for country H inflation and output, in a permanent liquidity trap are

$$\widehat{\Pi^B_{H,t}} = -0.0074 - 1.070\,\widehat{r_{H,t}} = -0.0074 + 0.05\,\widehat{\theta_{H,t}} - 0.03\widehat{G_{H,t}} - 0.05\,\widehat{\Psi_{H,t}} - 0.003\widehat{\Psi_{F,t}},$$

$$\widehat{Y_{H,t}^{B}} = -0.0001 + 1.02\,\widehat{\theta_{H,t}} + 0.49\,\widehat{G_{H,t}} - 0.08\,\widehat{\Psi_{H,t}} + 0.06\,\widehat{\Psi_{F,t}}.$$

Thus, a 1% percent increase in country k productivity raises domestic (gross) inflation by 0.05% (this corresponds to a rise of the annualized inflation rate by 0.2 percentage points); while a 1% increase in government purchases lowers gross inflation by 0.03%. [15] The government purchases multiplier (effect on output) is 0.49. Although the rise in government purchases lowers inflation, the government purchases multiplier is positive, because a rise in government purchases lowers consumption, which raises labor supply. Country H inflation and output do not depend on Foreign productivity or Foreign government purchases, in the model version considered here. This is due to the fact that the Home Euler-Phillips equation only involves Home inflation, and that the Home natural real interest rate does not depend on Foreign productivity or Foreign government purchases, as discussed in Sect. 2.6.

By contrast, Home consumption and the terms of trade depend on *both* countries' productivity and government purchases shocks. Also, Home consumption and the terms of trade depend on the Foreign ZLB regime, but quantitatively the effect of the Foreign ZLB regime is negligible. Let $\widehat{C_{H,t}^{BB}}$ and $\widehat{q_t^{BB}}$ denote country H consumption and the terms of trade when both countries are in a permanent liquidity trap. For ρ=0.95, we find:

$$\widehat{C_{H,t}^{BB}} = -0.0001 + 0.88\cdot\widehat{\theta_{H,t}} + 0.13\cdot\widehat{\theta_{F,t}} - 0.44\cdot\widehat{G_{H,t}} - 0.07\cdot\widehat{G_{F,t}} + 0.16\cdot\widehat{\Psi_{H,t}} - 0.18\cdot\widehat{\Psi_{F,t}}\,,$$

$$\widehat{q_t^{BB}} = -1.02\cdot(\widehat{\theta_{H,t}} - \widehat{\theta_{F,t}}) + 0.51\cdot(\widehat{G_{H,t}} - \widehat{G_{F,t}}) + 0.88\cdot(\widehat{\Psi_{H,t}} - \widehat{\Psi_{F,t}})\,.$$ [16]

Note that shock responses of output, consumption and terms of trade, in a liquidity trap are similar to the responses that obtain in flex-prices world (see (18)). This reflects the muted

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[15] The restriction $\gamma_\pi \widehat{\Pi_{H,t}^{B}} \leq -(\Pi-\beta)/\Pi$ requires upper bound restrictions on productivity and lower bound restrictions on government purchases and the preference shock. For example, if productivity and the preference shifter take steady state values, then $\widehat{G_{H,t}} \geq -9\%$ has to hold: when government purchases fall below this lower bound, then the inflation rate rises to a level which is such that the Taylor rule prescribes a strictly positive nominal interest rate, which violates (21).

[16] Denoting by $\widehat{C_{H,t}^{BS}}$ and $\widehat{q_t^{BS}}$ Home consumption and the terms of trade when H is in a permanent liquidity trap, while country F has a permanently slack ZLB constraint, we find

$$\widehat{C_{H,t}^{BS}} = -0.0001 + 0.88\cdot\widehat{\theta_{H,t}} + 0.13\cdot\widehat{\theta_{F,t}} - 0.44\cdot\widehat{G_{H,t}} - 0.06\cdot\widehat{G_{F,t}} + 0.16\cdot\widehat{\Psi_{H,t}} - 0.17\cdot\widehat{\Psi_{F,t}}\,;$$

$$\widehat{q_t^{BS}} = 0.0001 - 1.02\cdot\widehat{\theta_{H,t}} + 0.97\cdot\widehat{\theta_{F,t}} + 0.51\cdot\widehat{G_{H,t}} - 0.49\cdot\widehat{G_{F,t}} + 0.89\cdot\widehat{\Psi_{H,t}} - 0.84\cdot\widehat{\Psi_{F,t}}\,.$$

Thus, the decision rules for $\widehat{C_{H,t}^{BS}}$ and $\widehat{q_t^{BS}}$ are very similar to the decision rules for $\widehat{C_{H,t}^{BB}}$ and $\widehat{q_t^{BB}}$.

response of inflation to persistent fundamental shocks, under sticky prices. In equilibrium, inflation is a function of the natural real interest rate; persistent fundamental shocks have a muted effect on the natural real interest rate (as the latter depends on the expected future *change* of the fundamentals), which helps to understand the weak effect of these shocks on inflation. As pointed out above (Sect. 2.7), if inflation responses to exogenous shocks are sufficiently muted in a sticky-prices world, the transmission of those shocks to real activity resembles transmission under flexible prices.

3.2.2. Permanently slack ZLB constraint

I next consider an equilibrium in which country Home stays forever away from the ZLB, so that $\widehat{\Pi_{H,t}}=\widehat{\Pi^S_{H,t}}\ \ \forall t$ (see (20)). Then Home inflation is governed by the following Euler-Phillips equation (from (16)):

$$\gamma_\pi \widehat{\Pi^S_{H,t}} = -\tfrac{1}{\kappa}\widehat{\Pi^S_{H,t}} + (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi^S_{H,t+1}} - \tfrac{\beta}{\kappa}E_t\widehat{\Pi^S_{H,t+2}} + \widehat{r_{H,t}}.$$

Substitution of decision rule (20) into this equation shows that the coefficients of the decision rule are

$$\mu^S=0 \text{ and } \lambda^S=-(\kappa/\beta)/\{\Gamma(\rho)-\gamma_\pi\cdot(\kappa/\beta)\}.$$

$\gamma_\pi>1$ (Taylor principle) implies that $\Gamma(\rho)-\gamma_\pi\cdot(\kappa/\beta)<0\ \ \forall\ 0\leq\rho\leq 1$, and so $\lambda^S>0$: when the ZLB is always slack, then a rise in the natural real interest rate triggers a rise in the inflation rate, and thus the nominal interest rate increases.[17] Away from the ZLB, a rise in Home productivity (which reduces the Home natural interest rate) lowers Home inflation, while positive preference and government purchases shocks raise inflation. For ρ=0.95, the decision rules for Home inflation and output, under a permanently slack ZLB constraint are

$$\widehat{\Pi^S_{H,t}} = 1.77\,\widehat{r_{H,t}} = -0.09\,\widehat{\theta_{H,t}} + 0.04\,\widehat{G_{H,t}} + 0.08\,\widehat{\Psi_{H,t}} + 0.006\,\widehat{\Psi_{F,t}},$$

$$\widehat{Y^S_{H,t}} = 0.97\,\widehat{\theta_{H,t}} + 0.51\,\widehat{G_{H,t}} - 0.04\,\widehat{\Psi_{H,t}} + 0.07\,\widehat{\Psi_{F,t}}.$$

[17] Inflation in regime with a permanently slack ZLB constraint has to satisfy the restriction $\gamma_\pi\widehat{\Pi^S_{H,t}}\geq-\{(\Pi-\beta)/\Pi\}$ (see (21)), i.e. the inflation rate has to remain sufficiently high to ensure that the ZLB constraint does not binds. This restriction requires $\widehat{r_{H,t}}\geq-(1/\lambda^S)(\Pi-\beta)/\Pi$, where the right-hand side is strictly negative; thus the natural rate cannot drop too much.

Thus, although inflation responses to shocks are *qualitatively* different than in the permanent liquidity trap, we see that inflation responses remain rather weak, due to high shock persistence. This helps to understand why *output* responses to shocks are similar across sticky-prices ZLB regimes and the flex-prices economy (see Sect. 3.2.1 and (18)). It can, however, be noted that, with a permanently slack ZLB constraint, output is slightly less responsive to domestic productivity shocks, but slightly more responsive to domestic government purchase shocks than in a permanent liquidity trap.

When both countries are in the regime with a permanently slack ZLB constraint, then decision rules for Home consumption and the terms of trade are:

$$\widehat{C_{H,t}^{SS}} = 0.85\cdot\widehat{\theta_{H,t}} + 0.13\cdot\widehat{\theta_{F,t}} - 0.42\cdot\widehat{G_{H,t}} - 0.06\cdot\widehat{G_{F,t}} + 0.20\cdot\widehat{\Psi_{H,t}} - 0.17\cdot\widehat{\Psi_{F,t}},$$

$$\widehat{q_t^{SS}} = -0.97\cdot(\widehat{\theta_{H,t}} - \widehat{\theta_{F,t}}) + 0.49\cdot(\widehat{G_{H,t}} - \widehat{G_{F,t}}) + 0.85\cdot(\widehat{\Psi_{H,t}} - \widehat{\Psi_{F,t}}).$$

Thus, the consumption and terms of trade equations too are similar to the ones that obtain in a permanent liquidity trap, and in the flex-prices economy.

When country Home has a permanently slack ZLB constraint, then its nominal interest rate, in % p.a. is given by:

$$400\cdot i_{H,t+1} = 3.01 - .36\,\widehat{\theta_{H,t}} + 0.18\,\widehat{G_{H,t}} + 0.33\,\widehat{\Psi_{H,t}} + 0.02\,\widehat{\Psi_{F,t}}$$

Thus, the nominal interest rate exhibits a muted response to business cycle shocks (e.g. a 1% productivity increase raises the interest rate by merely 36 basis points per annum). This also helps to understand why the output response is so similar across ZLB regimes.

3.2.3. Time-varying ZLB regimes

I now consider equilibria in which countries randomly switch between ZLB regimes, because of self-fulfilling switches in agents' inflation expectations. See Appendix A for detailed derivations of material in this Section.

For simplicity, I assume that the ZLB regime is independent across countries, and independent of Home and Foreign fundamental forcing variables. Assume that the ZLB regime of country k=H,F follows a first-order Markov chain. Denote k's ZLB regime at date *t* by $z_{k,t} \in \{B,S\}$ where $z_{k,t}=B$ means that the ZLB constraint binds at date *t* in country k (so that decision rule (19) applies) while $z_{k,t}=S$ indicates that the ZLB constraint is slack (and decision rule (20) applies). Let the transition probabilities between ZLB regimes be $p_{ij} \equiv \text{Prob}(z_{k,t+1}=j | z_{k,t}=i)$ for $i,j \in \{\text{B;S}\}$, with $0 \le p_{ij} \le 1$ and $p_{iB}+p_{iS}=1$, and let $\Phi \equiv \begin{bmatrix} p_{BB} & p_{BS} \\ p_{SB} & p_{SS} \end{bmatrix}$ be the matrix of transition probabilities, and define $\widetilde{\Phi} \equiv \Phi \cdot \Phi$. Let $\mu \equiv [\mu^B; \mu^S]$ and $\lambda \equiv [\lambda^B; \lambda^S]$ denote 2x1 vectors that, respectively, include the intercepts and the slopes of the inflation decision rules (19),(20). Expected date t+1 inflation, conditional on the ZLB state and the natural real interest rate realized at date *t* is then:

$$E(\widehat{\Pi_{k,t+1}} | z_{k,t}=B, \widehat{r_{k,t}}) = \Phi_{1,\bullet}\{\mu + \lambda\rho\widehat{r_{k,t}}\},\ E(\widehat{\Pi_{k,t+1}} | z_{k,t}=S, \widehat{r_{k,t}}) = \Phi_{2,\bullet}\{\mu + \lambda\rho\widehat{r_{k,t}}\},$$

$$E(\widehat{\Pi_{k,t+2}} | z_{k,t}=B, \widehat{r_{k,t}}) = \widetilde{\Phi}_{1,\bullet}\{\mu + \lambda\rho^2\widehat{r_{k,t}}\},\ E(\widehat{\Pi_{k,t+2}} | z_{k,t}=S, \widehat{r_{k,t}}) = \widetilde{\Phi}_{2,\bullet}\{\mu + \lambda\rho^2\widehat{r_{k,t}}\}, \qquad (24)$$

where $\Phi_{i,\bullet}$ and $\widetilde{\Phi}_{i,\bullet}$ with i=1,2 denote the i-th rows of matrices Φ and $\widetilde{\Phi}$, respectively.

An equilibrium with recurrent liquidity traps in country k=H,F is defined by decision rule coefficients μ,λ and transition probabilities $0<p_{SS}, p_{BB}<1$ such that inequalities (21) are satisfied, and the Euler-Phillips equation (16) holds, in each ZLB regime:

$$-(\Pi-\beta)/\Pi = -\tfrac{1}{\kappa}\{\mu^B+\lambda^B\widehat{r_{k,t}}\} + (1+\tfrac{1+\beta}{\kappa})\Phi_{1,\bullet}\{\mu+\lambda\rho\widehat{r_{k,t}}\} - \tfrac{\beta}{\kappa}\widetilde{\Phi}_{1,\bullet}\{\mu+\lambda\rho^2\widehat{r_{k,t}}\} + \widehat{r_{k,t}}, \qquad (25)$$

$$\gamma_\pi\{\mu^S+\lambda^S\widehat{r_{k,t}}\} = -\tfrac{1}{\kappa}\{\mu^S+\lambda^S\widehat{r_{k,t}}\} + (1+\tfrac{1+\beta}{\kappa})\Phi_{2,\bullet}\{\mu+\lambda\rho\widehat{r_{k,t}}\} - \tfrac{\beta}{\kappa}\widetilde{\Phi}_{2,\bullet}\{\mu+\lambda\rho^2\widehat{r_{k,t}}\} + \widehat{r_{k,t}}. \qquad (26)$$

Equations (25) and (26) are, respectively, the country k Euler-Phillips equation if the ZLB constraint binds and if it is slack, at date *t*. Stacking (25) and (26) gives:

$$\begin{bmatrix} -(\Pi-\beta)/\Pi \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\kappa} & 0 \\ 0 & -\gamma_\pi - \frac{1}{\kappa} \end{bmatrix} \cdot \{\mu+\lambda\widehat{r_{k,t}}\} + (1+\tfrac{1+\beta}{\kappa})\Phi\cdot\{\mu+\lambda\rho\widehat{r_{k,t}}\} - \tfrac{\beta}{\kappa}\widetilde{\Phi}\cdot\{\mu+\lambda\rho^2\widehat{r_{k,t}}\} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \widehat{r_{k,t}}. \quad (27)$$

(27) holds for arbitrary values of the real natural interest rate $\widehat{r_{k,t}}$ iff

$$\mu = \left\{ \begin{bmatrix} -\frac{1}{\kappa} & 0 \\ 0 & -\gamma_\pi - \frac{1}{\kappa} \end{bmatrix} + (1+\tfrac{1+\beta}{\kappa})\Phi - \tfrac{\beta}{\kappa}\widetilde{\Phi} \right\}^{-1} \begin{bmatrix} -(\Pi-\beta)/\Pi \\ 0 \end{bmatrix}$$

$$\text{and } \lambda = \left\{ \begin{bmatrix} -\frac{1}{\kappa} & 0 \\ 0 & -\gamma_\pi - \frac{1}{\kappa} \end{bmatrix} + (1+\tfrac{1+\beta}{\kappa})\Phi\rho - \tfrac{\beta}{\kappa}\widetilde{\Phi}\rho^2 \right\}^{-1} \begin{bmatrix} -1 \\ -1 \end{bmatrix}. \qquad (28)$$

The following condition ensures that the inequality constraints (21) hold for values of $\widehat{r_{k,t}}$ sufficiently close to zero:

$$\gamma_\pi \mu^B < -(\Pi-\beta)/\Pi < \gamma_\pi \mu^S. \qquad (29)$$

The decision rule coefficients μ,λ associated with a *given* transition probability matrix Φ are (generically) unique. By contrast, Φ is not uniquely pinned down by the Euler-Phillips equation (16) and by the restriction (29), but the existence of an equilibrium with time-varying ZLB regimes requires probabilities p_{SS} and p_{BB} close to unity.[18] This implies that the model of expectations-driven liquidity traps is well-suited for generating long-lasting liquidity traps—in fact that model *requires* a high expected duration of liquidity traps.

The following numerical simulations of the model variant with time-varying ZLB regimes assume $p_{SS}=p_{BB}=0.95$, which corresponds to an expected regime duration of 20 periods. Then, the decision rules for Home inflation and output in the regime with a binding ZLB constraint ('B') are

$$\widehat{\Pi^B_{H,t}} = -0.0080 - 1.36\,\widehat{r_{H,t}} = -0.0080 + 0.07\,\widehat{\theta_{H,t}} - 0.03\widehat{G_{H,t}} - 0.064\,\widehat{\Psi_{H,t}} - 0.004\,\widehat{\Psi_{F,t}}\,,$$

$$\widehat{Y^B_{H,t}} = -0.0022 + 1.06\,\widehat{\theta_{H,t}} + 0.47\,\widehat{G_{H,t}} - 0.12\,\widehat{\Psi_{H,t}} + 0.06\,\widehat{\Psi_{F,t}}.$$

The corresponding decision rules in the regime with a slack ZLB ('S') are

$$\widehat{\Pi^S_{H,t}} = -0.0011 + 1.28\,\widehat{r_{H,t}} = -0.0011 - 0.06\,\widehat{\theta_{H,t}} + 0.03\,\widehat{G_{H,t}} + 0.060\,\widehat{\Psi_{H,t}} + 0.004\,\widehat{\Psi_{F,t}},$$

$$\widehat{Y^S_{H,t}} = 0.0020 + 0.94\,\widehat{\theta_{H,t}} + 0.53\,\widehat{G_{H,t}} - 0.01\,\widehat{\Psi_{H,t}} + 0.07\,\widehat{\Psi_{F,t}}\,.$$

As ZLB regimes are persistent, it is not surprising that the decision rules are similar to the ones that obtain when each regime is permanent (see Sect. 3.2.1 and 3.2.2). Also, note again that the output decision rules are quite close to the flex-prices decision rules (see (18)). The same holds for the decision rules describing the terms of trade and consumption (see simulated shock responses below). It remains true that, in a liquidity trap, a positive supply shock raises domestic inflation, while a positive aggregate demand shock lowers domestic inflation. Importantly, the responses of output to productivity and government purchases shocks are again similar across the ZLB regimes. The government purchases multiplier is close to 0.5 in both ZLB regimes.

The effect of a ZLB regime shift on inflation and output depends on the level of the forcing variables. Note that $\widehat{Y^B_{H,t}} - \widehat{Y^S_{H,t}} = -0.0042 + 0.12\,\widehat{\theta_{H,t}} - 0.06\,\widehat{G_{H,t}} - 0.11\,\widehat{\Psi_{H,t}} - 0.01\,\widehat{\Psi_{F,t}}$.

[18] This is also noted by Arifovic et al. (2018), in a closed economy model. When p_{SS} and p_{BB} are *not* sufficiently close to unity, then the vector μ defined in (28) violates the inequalities (29). E.g., if agents believe that a liquidity trap is transient, then inflation is too high during a liquidity trap (as agents expect a rapid return to the 'slack-ZLB' regime), i.e. a liquidity trap is impossible.

Thus, entry into a liquidity trap has a detrimental effect on domestic output; the detrimental effect is greater when productivity is low and government purchases are high.

3.3. Simulated shock responses: expectations-driven ZLB regimes

Table 1 reports shock responses for the baseline New Keynesian model with expectations-driven ZLB regimes. ZLB regime persistence is set at $p_{SS}=p_{BB}=0.95$; the autocorrelation of the forcing variables is ρ=0.95. 1% innovations to Home productivity, Home government purchases and to the Home preference shifter (Ψ) are considered. Responses 0 and 12 periods after the shock are reported; see Column labelled 'Horizon'.[19] Responses of Home and Foreign nominal interest rates, inflation, output and consumption are shown, as well as responses of the Home terms of trade, the nominal exchange rate and Home net exports (normalized by GDP). All responses pertain to simulation runs *without* ZLB regime change. Panel (a) of Table 1 shows responses that obtain when both countries are in an expectations-driven liquidity trap, while Panel (b) assumes that the ZLB constraint is slack in both countries. A positive Home productivity (government purchases) shock triggers an interest rate cut (increase) when Home is away from the ZLB. However, shock responses of output, consumption and the real exchange rate are qualitatively and quantitatively similar across ZLB regimes. In both regimes, a positive productivity shock raises domestic and foreign consumption, and it triggers a nominal and real depreciation of the currency of the country receiving the shock; net exports and Foreign output are unaffected by the shock. A positive shock to government purchases lowers domestic and foreign consumption and it triggers a nominal and real exchange rate appreciation.

4. Fundamentals-driven liquidity traps

As discussed in the Introduction, an extensive literature has considered "fundamentals-driven" liquidity traps induced by large shocks to household preferences (or other fundamentals) that sharply reduce aggregate demand and push the nominal interest rate to the ZLB. The literature shows that, in a fundamentals-driven liquidity trap, fiscal spending multipliers can be markedly higher than when the ZLB does not bind; also, a positive technology shock can trigger an output

[19] In the model with expectations-driven liquidity traps, the dynamic shock responses of all variables (except the nominal exchange rate) decay geometrically with factor ρ (for a simulation run without change of ZLB regime). Thus it seems unnecessary to show more detailed response trajectories. By contrast, for fundamentals-driven liquidity traps, more detailed responses will be reported, as shock responses do not exhibit geometric decay.

contraction (e.g., Eggertsson and Woodford (2003)). Many influential studies on liquidity traps in open economies have likewise considered fundamentals-driven liquidity traps (see references in Sect. 1).

For comparison purposes with the expectations-driven liquidity traps analyzed in the previous Section, I now discuss a fundamentals-driven liquidity trap, in the two-country model used above. Following Blanchard et al. (2016), I consider liquidity traps driven by unanticipated one-time shocks at some date t=0 that depress the natural real interest rate below its steady level, so that $\widehat{r_{k,0}}<0$. Except for shocks at date t=0, there are no random disturbances. Thus the economy evolves deterministically (perfect foresight), after t=0. For given initial adverse shocks, there exists a unique deterministic equilibrium in which the liquidity trap ends permanently after a finite time span.[20]

As there are no exogenous innovations after date t=0, the natural real interest rate in country k=H,F at $t\geq0$ is: $\widehat{r_{k,t}}=\rho^t\cdot\widehat{r_{k,0}}$, where $0\leq\rho<1$ is the autocorrelation of the exogenous forcing processes and of the natural real interest rate.

In a deterministic equilibrium *without* ZLB constraint, the (gross) inflation rate and the (gross) nominal interest rate (expressed in 'hatted' form, i.e. as deviations from steady state) of country k=H,F at dates $t\geq0$ would be

$$\widehat{\Pi^*_{k,t}}=\lambda^S\rho^t\,\widehat{r_{k,0}} \text{ and } \widehat{1+i^*_{k,t+1}}=\gamma_\pi\lambda^S\rho^t\,\widehat{r_{k,0}}, \tag{30}$$

respectively, where $\lambda^S>0$ is the decision rule coefficient (for inflation) in a regime with a permanently slack ZLB constraint (see Sect. 3.2.2).[21] A fundamentals-driven liquidity trap occurs in country k when the country's unconstrained nominal interest rate is negative at date t=0, i.e. when

$$\widehat{1+i^*_{k,1}}<-(\Pi-\beta)/\Pi. \tag{31}$$

[20] In the model here, the Euler-Phillips equation (16) does not include lagged endogenous state variables. As shown by Holden (2016, 2019), this ensures that an equilibrium featuring eventual permanent exit from the liquidity trap is unique; models with endogenous state variables may have multiple deterministic equilibria that eventually escape from the liquidity trap.

[21] In a world without ZLB constraint, the monetary policy rule (5) is replaced by: $1+i_{k,t+1}=\Pi/\beta+(\gamma_\pi/\beta)\cdot(\Pi_{k,t}-\Pi)$ which implies $\widehat{1+i_{k,t+1}}=\gamma_\pi\,\widehat{\Pi_{kt}}$ for k=H,F.

This inequality holds when the country k real natural rate at date t=0 is sufficiently low. Assume that (31) applies, and let T_k^* be the smallest value of t≥0 for which the unconstrained nominal rate becomes non-negative again, i.e. $\widehat{1+i^*_{k,t+1}} \geq -(\Pi-\beta)/\Pi$. Thus,

$$\widehat{1+i^*_{k,T_k^*}} < -(\Pi-\beta)/\Pi \quad \text{and} \quad \widehat{1+i^*_{k,T_k^*+1}} \geq -(\Pi-\beta)/\Pi.$$

A fundamentals-driven liquidity trap equilibrium has the property that the ZLB constraint binds in country k until period T_k^*-1, and that the ZLB does not bind in $t \geq T_k^*$. Thus, $\widehat{\Pi_{k,t}}=\widehat{\Pi^*_{k,t}}$ and $\widehat{1+i_{k,t+1}}=\widehat{1+i^*_{k,t+1}}$ hold for $t \geq T_k^*$ (where $\widehat{\Pi^*_{k,t}}$ and $\widehat{1+i^*_{k,t+1}}$ are defined in (30)). In periods $t < T_k^*$, the country k nominal interest rate is zero, i.e. $\widehat{1+i_{k,t+1}}=-(\Pi-\beta)/\Pi$. From the Euler-Phillips equation (16), we see that country k inflation at dates $0 \leq t < T_k^*$ has to obey the condition $-(\Pi-\beta)/\Pi = -\frac{1}{\kappa}\widehat{\Pi_{k,t}} + (1+\frac{1+\beta}{\kappa})\widehat{\Pi_{k,t+1}} - \frac{\beta}{\kappa}\widehat{\Pi_{k,t+2}} + \widehat{r_{k,t}}$. Thus,

$$\widehat{\Pi_{k,t}} = \kappa(\Pi-\beta)/\Pi + (1+\beta+\kappa)\widehat{\Pi_{k,t+1}} - \beta\widehat{\Pi_{k,t+2}} + \kappa\widehat{r_{k,t}} \quad \text{for } 0 \leq t < T_k^*. \tag{32}$$

Iterating (32) backward in time allows to compute country k inflation at dates $0 \leq t < T_k^*$.[22] Importantly, the largest root of the "backward" inflation iteration equation (32) exceeds unity. Thus, the backward inflation loop is explosive (as noted by Cochrane (2017) and Maliar and Taylor (2019), in related models). This implies that the response of inflation at t=0 to the shock that triggers a fundamentals-driven liquidity trap can be large, as confirmed by the simulations below. Also, shocks to the natural real interest rate that induce small changes in the inflation rate in period T_k^*, i.e. when country k emerges from the liquidity trap, may have a big effect on inflation, and thus on output, at the *start* of the liquidity trap. This explains, for example, why fiscal multipliers in a fundamentals-driven liquidity trap can be very large (see below).

Table 2 reports dynamic shock responses, when *both* countries are in a fundamentals-driven liquidity trap. (In Sect. 5, I also consider a fundamentals-driven liquidity trap in just one country.) All preference, technology and price stickiness parameters are set at the values used in previous Sections (thus the autocorrelation of the exogenous forcing processes is again set at ρ=0.95).

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[22] Inflation in T^*-1 (last period of the liquidity trap) is $\widehat{\Pi_{H,T^*-1}}=\kappa(\Pi-\beta)/\Pi+(1+\beta+\kappa)\widehat{\Pi^*_{H,T^*}}-\beta\widehat{\Pi^*_{H,T^*+1}}+\kappa\widehat{r_{H,T^*-1}}$ etc.

Following Blanchard et al. (2016), I consider a baseline fundamentals-driven liquidity trap scenario that lasts 12 quarters. That baseline scenario is brought about by unanticipated one-time -9.89% identical innovations to the Home and Foreign preference shifters (Ψ) at t=0 that depresses the natural real interest rate in both countries by 46 basis points, on impact. Panel (a) in Table 2 reports the dynamics of the two countries, under the baseline liquidity trap scenario.[23] Panel (b) shows dynamic responses that obtain when positive 1% date t=0 innovations to Home productivity, Home government purchases and the Home preference shifter (Ψ) are *added* to the baseline liquidity trap scenario; those dynamic responses are shown as deviations from the baseline liquidity trap scenario.

The simulations in Table 2 highlight important qualitative and quantitative transmission differences across expectations- and fundamentals-driven liquidity traps.

The baseline fundamentals-driven liquidity trap scenario features a sharp, but short-lived, contraction in inflation, output and consumption. Inflation and output drop by 26 ppt p.a. and by 13%, respectively, on impact. The effect (in absolute value) of a fundamentals-driven liquidity trap is strongly increasing in the duration of the trap, e.g., when the fundamentals-driven liquidity trap is lengthened by only 3 quarters (to 15 quarters), the initial drops in inflation and output are 92 ppt p.a. and 43%, respectively. That very strong sensitivity of inflation and output to the length of the fundamentals-driven liquidity trap is unappealing. The model variant with expectations-driven ZLB regimes is better suited for generating persistent liquidity traps. In that model variant, assuming more persistent liquidity traps (by raising ZLB regime persistence $p_{SS}=p_{BB}$ above the baseline 0.95 value; see Sect. 3) has a minor effect on inflation and output.

As a unit trade elasticity is assumed in the present model version, it predicts that country-specific productivity and government purchases shocks only affect inflation and output in the country that receives the shock (see Panel (b)). By contrast, preference shocks induce international spillovers.

In a fundamentals-driven liquidity trap, a Home productivity increase has a strong negative effect on Home inflation and output; on impact, a 1% Home productivity innovation lowers Home inflation and output by 32 ppt and 15%, respectively (see Panel (b) in Table 2). Intuitively, a persistent productivity increase lowers Home inflation, when the country emerges

[23] To save space, I only report responses 0, 5 and 12 periods after the shock (period 12 corresponds to the first period after exit from the liquidity trap).

from the liquidity trap. The explosive backward iteration described above (see (32)) then implies a strong fall in Home inflation, at the start of the liquidity trap. The sharp initial contraction in Home inflation is accompanied by a strong contraction in Home output and in Home consumption. Due to full risk sharing, the Home consumption contraction is associated with a strong appreciation of the Home nominal and real exchange rates (and an improvement in the Home terms of trade).

A positive innovation to Home government purchases has a strong positive initial effect on Home inflation and output. This is accompanied by a sizable *depreciation* of the Home real exchange rate and a *rise* in Home consumption. The fiscal spending multiplier is big (9.2, on impact).

The previous literature on fundamentals-driven liquidity traps has highlighted non-standard (topsy-turvy) output responses to productivity shocks, as well as the large fiscal multipliers in fundamentals-driven liquidity traps (e.g., Eggertsson (2010), Eggertsson and Krugman (2012)). However, the "unorthodox" response of the real exchange rate to productivity and fiscal shocks has apparently not previously been noticed.

The next Section considers a model version with a non-unitary trade elasticity. That version gives rise to international spillovers that can be qualitatively different and much larger in fundamentals-driven liquidity traps than in expectations-driven traps.

5. Sensitivity analysis

5.1. Trade elasticity larger than unity

I now replace the Cobb-Douglas Home consumption aggregator used in the baseline model (see Sect. 2.1) by the CES aggregator $C_{H,t} \equiv \{\xi^{1/\phi}(Y_{H,t}^{H})^{(\phi-1)/\phi} + (1-\xi)^{1/\phi}(Y_{H,t}^{F})^{(\phi-1)/\phi}\}^{\phi/(\phi-1)}$ where ϕ (with $\phi>0$, $\phi\neq 1$) is the substitution elasticity between domestic and imported intermediates (trade elasticity). As before, $\frac{1}{2}<\xi<1$ is assumed (consumption home bias). The Cobb-Douglas aggregator assumed in Sect. 3-4 implies a unit substitution elasticity, $\phi=1$.[24] Appendix B derives the solution for $\phi\neq 1$. A key finding is that, for $\phi\neq 1$, a country's Euler-Phillips equation involves domestic and *foreign* inflation, and that the natural real interest rate depends on domestic and foreign productivity and government purchases.

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[24] The Cobb-Douglas aggregator is the limit of the CES aggregator as $\phi \to 1$.

Many open economy macro models assume $\phi>1$. The following model simulations set $\phi=1.5$. That value is consistent with time-series estimates of the price elasticity of aggregate trade flows (Kollmann (2001)), and it is, e.g., used in the canonical two-country international Real Business Cycle model of Backus et al. (1994).[25]

As shown in Appendix B, a flex-prices model predicts that a positive shock to Home productivity raises Home net exports and lowers Foreign output when $\phi>1$; intuitively, the terms of Home trade deterioration triggered (under flexible prices) by a rise in Home productivity induces stronger expenditure-switching away from Foreign goods, when $\phi>1$ (compared to the baseline case $\phi=1$), which raises Home net exports and lowers Foreign output. By contrast, a positive shock to Home government purchases lowers Home net exports and raises Foreign output when $\phi>1$, which reflects stronger expenditure switching towards Foreign goods, in response to the Home terms of trade appreciation triggered by the fiscal shock. As discussed below, the same qualitative international transmission effects obtain in an expectations-driven liquidity trap, under sticky prices, if shocks are persistent.

Preference shocks already generate international spillovers when a unit trade elasticity is assumed ($\phi=1$). Qualitatively, responses to preference shocks do not change when $\phi>1$ is assumed. To save space, the simulations and discussions of the economy with $\phi>1$ thus focus on productivity and government purchases shocks.

Table 3 reports impact responses to 1% positive innovations to Home productivity and to Home government purchases, in a sticky-prices model version with expectations-driven ZLB regimes, for trade elasticity $\phi=1.5$; all other parameters are unchanged compared to Table 1.[26] Panel (a) shows responses that obtain when both countries are in a liquidity trap, Panel (b) assumes that only country Home is in a liquidity trap, and Panel (c) assumes that the ZLB constraint is slack in both countries. Shock responses of Home and Foreign output and consumption, and of the terms of trade and net exports are similar to the responses predicted by a flex-prices model, and that irrespective of the (Home and Foreign) ZLB regimes (flex-prices decision rules for $\phi=1.5$ are reported in Appendix B). With $\phi=1.5$, a Home productivity increase raises Home next exports and lowers Foreign output. A rise in Home government purchases

[25] Values of ϕ in the range of 1.5 are also produced by econometric estimates of multi-county structural macro models; see, e.g., Giovannini et al. (2019) and Kollmann et al. (2015).

[26] Thus, the autocorrelation of fundamental shocks is again set at $\rho=0.95$, and the probability of remaining in the same ZLB regime is $p_{SS}=p_{BB}=.95$.

lowers Home net exports; it raises Foreign output, while reducing Foreign consumption. When ϕ=1.5, the domestic and foreign transmission of Home shocks depends on the Foreign ZLB regime, but the effect of the Foreign ZLB regime is very weak (see Panel (c)).

I next turn to shock responses in a sticky-prices model version with fundamentals-driven liquidity traps, for trade elasticity ϕ=1.5; see Table 4. [27] Those shock responses again indicate important qualitative and quantitative differences compared to expectations-driven liquidity traps. Panel (a) of Table 4 considers the case where both countries are in a fundamentals-driven liquidity trap (induced by identical negative Home and Foreign preference shocks). The effects of innovations to Home productivity and government purchases on Home inflation, output and the terms of trade are similar to the ones predicted under ϕ=1. E.g., it is again found that, in a fundamentals-driven liquidity trap, a rise in Home productivity triggers a strong transitory *fall* in Home inflation and output, and a marked transitory *improvement* in the Home terms of trade, while a rise in Home government purchases triggers a strong rise in Home inflation and output, and a *deterioration* of the Home terms of trade. However, when the trade elasticity exceeds unity, the rise in Home productivity *reduces* Home net exports and it *raises* Foreign output (while these variables are unaffected under a unit trade elasticity), when both countries are in a fundamentals-driven liquidity trap. The intuition is that, with the higher trade elasticity, the *improvement* of the Home terms of trade (triggered by the Home productivity shock) induces stronger expenditure switching towards Foreign goods. By the same logic, the rise in Home government purchases *raises* Home net exports, and *lowers* Foreign output, due to a stronger expenditure switching effect towards Home goods, induced by the worsening of the Home terms of trade (triggered by the shock). Hence, with ϕ=1.5, international spillover effects are *opposite* of those predicted in an expectations-driven liquidity trap.

Panel (b) of Table 4 considers a case in which only country Home is in a fundamentals-drive liquidity trap, while the Foreign ZLB constraint does not bind (the Home liquidity trap is brought about by a large negative Home preference shock). In that environment, a Home productivity increase again leads to a sharp Home terms of trade improvement, a worsening of Home net exports and a rise in Foreign output. By contrast, a *Foreign* productivity increase

[27] Table 4 is a counterpart to Table 2, for ϕ=1.5. Thus, again, baseline fundamentals-driven liquidity trap scenarios are assumed that are triggered by large negative preference shocks and last 12 periods. Dynamic responses to 1% productivity and government purchases innovations are again reported as differences compared to the baseline liquidity trap scenarios.

worsens the Foreign terms of trade and improves Foreign net exports (i.e. Home terms of trade improve, and Home net exports fall), and Home output falls.

5.2. Less persistent shocks

The previous simulations assumed persistent shocks (ρ=0.95), in line with empirical autocorrelations, and with autocorrelations typically assumed in macro models. In an expectations-driven liquidity trap, a transient productivity increase lowers the inflation rate, while a transient increase in government purchases raises the inflation rate (see discussion in Sec. 3.2.1). These predicted inflation responses are opposite to the ones that obtain under persistent shocks. Table 5 shows impact effects of Home productivity and government purchases innovations, for a sticky-prices model version with expectations-driven ZLB regimes, assuming a ρ=0.5 shock autocorrelation; the trade elasticity is set at ϕ=1.5, to allow for international output spillovers. All other parameters are the same as in Table 3.

Transitory shocks have a bigger effect on the natural real interest rate, and they trigger a greater (absolute) inflation response, on impact. For example, a 1% Home productivity innovation now lowers Home inflation by 1.89 ppt, on impact, when both countries are in an expectations-driven liquidity trap (Panel (a), Table 5); the strong Home inflation decrease is accompanied by a *contraction* in Home output and consumption, and by an *improvement* of the Home terms of trade, a *fall* in Home net exports and a *rise* in Foreign output. The strong rise of Home inflation triggered by a transitory rise in Home government purchases implies that Home output rises markedly more, on impact, than in response to a persistent government purchases shock; a transitory Home fiscal shock deteriorates the Home terms of trade, raises Home net exports and *lowers* Foreign output. In an expectations-driven liquidity trap, responses to transient productivity shock differ thus qualitatively from the responses to persistent shocks discussed above, and they also deviate markedly from responses under flexible prices.

In fact, for ρ=0.5, shock responses under an expectations-driven liquidity trap are qualitatively and quantitatively similar to responses under a fundamentals-driven liquidity trap. This can be seen from Table 6, where a sticky-prices model with fundamentals-driven liquidity traps is considered, in which the autocorrelations of productivity and government purchases are set at ρ=0.5 (ϕ=1.5 is assumed). All other parameters are the same as in Table 4. [28] In fundamentals-driven liquidity traps, the responses to transitory productivity and government purchases shocks are weaker than responses to persistent shocks, but the qualitative features of shock responses are unchanged; e.g., it remains true that a positive Home productivity shock lowers Home output, improves the Home terms of trade, lowers Home net exports and raises Foreign output.

6. Conclusion

This paper has studied a New Keynesian model of a two-country world with a zero lower bound (ZLB) constraint for nominal interest rates. A floating exchange rate regime is assumed. The presence of the ZLB generates multiple equilibria. The two countries can experience recurrent liquidity traps induced by self-fulfilling domestic and foreign inflation expectations. These "expectations-driven" liquidity traps can be synchronized or unsynchronized across countries. The model of expectations-driven liquidity traps is well-suited for generating long-lasting liquidity traps. The domestic and international transmission of persistent fundamental business cycle shocks in an expectations-driven liquidity trap differs markedly (both qualitatively and quantitatively) from shock transmission in a fundamentals-driven liquidity trap. In an expectations-driven liquidity trap, persistent productivity and government purchases shocks trigger responses of real activity and the exchange rate that are similar to standard predicted responses that obtain when the ZLB does not bind. E.g., a persistent Home productivity increase raises Home output and depreciates the Home real exchange rate, both at the ZLB and away from the ZLB. For a trade elasticity greater than unity, the model with expectations-driven liquidity traps developed here predicts that a persistent rise in Home productivity raises Home net exports and lowers Foreign output, while a persistent rise in Home government purchases lowers Home net exports and raises Foreign output. These international spillover effects are *opposite* of those predicted in a fundamentals-driven liquidity trap.

[28] To facilitate comparison with the predicted shock responses shown in Table 4, Table 6 assumes the same fundamentals-driven liquidity trap scenarios as Table 4; thus, in both Tables, the baseline liquidity trap scenarios are induced by negative preference shocks whose autocorrelation is 0.95; the sole difference between the two Tables is that the autocorrelation of productivity and fiscal shocks is 0.95 in Table 4, compared to the 0.5 autocorrelation in Table 6.

References

Acharya, Sushant and Julien Bengui, 2018. Liquidity Traps, Capital Flows. Journal of International Economics 114, 276-298.

Alvarez, Luis, Emmanuel Dhyne, Marco Hoeberichts, Claudia Kwapil, Hervé Le Bihan, Patrick Lünnemann, Fernando Martins, Roberto Sabbatini, Harald Stahl, PhilipVermeulen and Jouko Vilmunen, 2006. Journal of the European Economic Association 4, 575-584.

Andrade, Philippe, Jordi Galí, Hervé Le Bihan and Julien Matheron, 2019. The Optimal Inflation Target and the Natural Rate of Interest. Brookings Papers on Economic Activity, Fall 2019, 173-230.

Andrade, Philippe, Jordi Galí, Hervé Le Bihan and Julien Matheron, 2020. Should the ECB Adjust its Strategy in the Face of a Lower Natural Rate of Interest? Working Paper

Arifovic, Jasmina, Stephanie Schmitt-Grohé and Martin Uribe, 2018. Learning to Live in a Liquidity Trap. Journal of Economic Dynamics & Control 89, 120-136.

Aruoba, Boragan, Pablo Cuba-Borda and Frank Schorfheide, 2018. Macroeconomic Dynamics Near the ZLB: a Tale of Two Countries. Review of Economic Studies 85, 87-118.

Backus, David, Patrick Kehoe, and Finn Kydland, 1994. Dynamics of the Trade Balance and the Terms of Trade: The J-Curve? American Economic Review 84, 84-103.

Backus, David, and Gregor Smith, 1993. Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods. Journal of International Economics 35, 297-316.

Badarau, Cristina and Ibrahima Sangaré, 2019. Exchange Rate Regimes in a Liquidity Trap. Journal of International Money and Finance 93, 55-80.

Balfoussia, Hiona, Harris Dellas, Dimitris Papageorgiou and Evangelia Vourvachaki, 2020. How Does the ZLB Affect the Properties of Alternative Exchange Rate Systems? Working Paper, Bank of Greece.

Barthélemy, Jean and Magali Marx, 2019. Monetary Policy Switching and Indeterminacy. Quantitative Economics 10, 352-385.

Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe, 2001a. The Perils of Taylor Rules. Journal of Economic Theory 96, 40-69.

Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe, 2001b. Monetary Policy and Multiple Equilibria. American Economic Review 91, 167-186.

Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe, 2002a. Avoiding Liquidity Traps. Journal of Political Economy 110, 535-563

Benhabib, Jess, Stephanie Schmitt-Grohé and Martín Uribe, 2002b. Chaotic Interest-Rate Rules. American Economic Review 92, 72-78.

Benigno, Gianluca and Luca Fornaro, 2018. Stagnation Traps. Review of Economic Studies 85, 1425-1470.

Blanchard, Olivier, Christopher Erceg and Jesper Lindé, 2016. Jump-Starting the Euro Area Recovery: Would a Rise in Core Fiscal Spending Help the Periphery? NBER Macroeconomics Annual 31, 103-182.

Bussière, Matthieu, Giovanni Callegari, Fabio Ghironi, Giulia Sestieri, Norihiko Yamano, 2013. Estimating Trade Elasticities: Demand Composition and the Trade Collapse of 2008-2009. American Economic Journal: Macroeconomics 5, 118-151.

Calvo, Guillermo, 1983. Staggered Prices in a Utility-Maximizing Framework. Journal of Monetary Economics 12, 383-398.

Cho, Seonghoon and Moreno, Antonio, 2011. The Forward Method as a Solution Refinement in Rational Expectations Models. Journal of Economic Dynamics & Control 35, 257-272.

Cho, Seonghoon, 2016. Sufficient Conditions for Determinacy in a Class of Markov-Switching Rational Expectations Models. Review of Economic Dynamics 21, 182-200.

Christiano, Larry, Martin Eichenbaum and Sergio Rebelo, 2011. When is the Government Spending Multiplier Large? Journal of Political Economy 119, 78-121.

Cochrane, John, 2017. The New Keynesian Liquidity Trap. Journal of Monetary Economics 92, 47-63.

Coenen, Günter, Carlos Montes-Galdon and Sebastian Schmidt, 2020. Macroeconomic Stabilisation and Monetary Policy Effectiveness in a Low-Interest-Rate Environment. Working Paper, ECB.

Cole, Harold and Maurice Obstfeld, 1991. Commodity Trade and International Risk Sharing. Journal of Monetary Economics 28, 3-24.

Cook, David and Michael Devereux, 2013. Sharing the Burden: Monetary and Fiscal responses to a World Liquidity Trap 5, 190-228.

Cook, David and Michael Devereux, 2016. Exchange Rate Flexibility Under the Zero Lower Bound. Journal of International Economics 101, 52–69.

Corsetti, Giancarlo, Eleonora Mavroeidi, Gregory Thwaites and Martin Wolf, 2018. Step Away from the Zero Lower Bound: Small Open Economies in a World of Secular Stagnation. Journal of International Economics 116, 88-102.

Davig. Troy and Eric Leeper, 2007. Generalizing the Taylor Principle. American Economic Review 97, 607-635.

Davig. Troy and Eric Leeper, 2010. Generalizing the Taylor Principle: Reply. American Economic Association 200, 618-624.

Eggertsson, Gauti and Michael Woodford, 2003. The Zero Bound on Interest Rates and Optimal Monetary Policy. Brookings Papers on Economic Activity 1:2003, 139-211.

Eggertsson, Gauti, 2010. The Paradox of Toil. Staff Report 433, Federal Reserve Bank of New York.

Eggertsson, Gauti and Paul Krugman, 2012. Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach. Quarterly Journal of Economics 127, 1469-1513.

Erceg, Christopher and Jesper Lindé, 2010. Asymmetric Shocks in a Currency Union with Monetary and Fiscal Handcuffs. NBER International Seminar on Macroeconomics 7, 95-136.

Erceg, Christopher, Zoltan Jakab and Jesper Lindé, 2020. Monetary Policy Strategies for the European Central Bank. Working Paper, IMF.

Farhi, Emmanuel and Ivan Werning, 2016. Fiscal Multipliers: Liquidity Traps and Currency Unions. In: Handbook of Macroeconomics (J. Taylor and H. Uhlig, eds.), Elsevier, Vol. 2B, 2417-2492.

Farhi, Emmanuel, Ricardo Caballero and Pierre-Olivier Gourinchas, 2020. Global Imbalances and Policy Wars at the Zero Lower Bound. CEPR Discussion Paper 14424.

Farmer, Roger, Daniel Waggoner and Tao Zha, 2010. Generalizing the Taylor Principle: Comment. American Economic Review 100, 608-17.

Farmer, Roger, Vadim Kharamov and Giovanni Nicolò, 2015. Solving and Estimating Indeterminate DSGE Models. Journal of Economic Dynamics & Control 54, 17-36.

Fornaro, Luca and Federica Romei, 2019. The Paradox of Global Thrift. American Economic Review 109, 3745-3779.

Fujiwara, Ippei and Kozo Ueda, 2013. The Fiscal Multiplier and Spillover in a Global Liquidity Trap. Journal of Economic Dynamics & Control 37, 1264-1283.

Gomez, Sandra, Pascal Jacquinot, Ricardo Mestre and Joao Sousa. 2015. Global Policy at the Zero Lower Bound in a Large-Scale DSGE Model. Journal of International Money and Finance 50, 134-153.

Giovannini, Massimo, Stefan Hohberger, Robert Kollmann, Marco Ratto, Werner Roeger and Lukas Vogel, 2019. Euro Area and U.S. External Adjustment: The Role of Commodity Prices and Emerging Market Shocks. Journal of International Money and Finance 94, 183-205.

Hicks, John, 1937. Mr. Keynes and the "Classics": A Suggested Interpretation. Econometrica 5, 147-159.

Holden, Tom, 2016. Computation of Solutions to Dynamic Models With Occasionally Binding Constraints. Working Paper.

Holden, Tom, 2019. Existence and Uniqueness of Solutions to Dynamic Models With Occasionally Binding Constraints. Working Paper, Deutsche Bundesbank.

Jeanne, Olivier, 2009. Global Liquidity Traps, Working Paper.

Jeanne, Olivier, 2010. La Trappe à Liquidité Globale. Revue Economique 61, 395-406.

Keynes, John Maynard, 1936. The General Theory of Employment, Interest and Money. London: Macmillan.

King, Robert and Sergio Rebelo, 1999. Resuscitating Real Business Cycles, in: Handbook of Macroeconomics (J. Taylor and M. Woodford, eds.), Elsevier, Vol. 1B, pp. 927-1007.

Kollmann, Robert, 1991. Essays on International Business Cycles. PhD Dissertation, Economics Department, University of Chicago.

Kollmann, Robert, 1995. Consumption, Real Exchange Rates and the Structure of International Asset Markets. Journal of International Money and Finance 14, 191-211.

Kollmann, Robert, 2001. The Exchange Rate in a Dynamic-Optimizing Business Cycle Model with Nominal Rigidities: a Quantitative Investigation. Journal of International Economics 55, 243-262.

Kollmann, Robert, 2002. Monetary Policy Rules in the Open Economy: Effects on Welfare and Business Cycles. Journal of Monetary Economics 49, 989-1015.

Kollmann, Robert, 2004. Welfare Effects of a Monetary Union. Journal of the European Economic Association 2, 289-301.

Kollmann, Robert, Marco Ratto, Werner Roeger, Jan in't Veld and Lukas Vogel, 2015. What Drives the German Current Account? And How Does it Affect Other EU Member States? Economic Policy 40, 47-93.

Kollmann, Robert, 2020. Liquidity Traps in a Monetary Union. CEPR Discussion Paper 15148.

Krugman, Paul, 1998. It's Baaack: Japan's Slump and the Return of the Liquidity Trap. Brookings Papers on Economic Activity 2:1998, 137-205.

Lubik, Thomas and Frank Schorfheide, 2003. Computing Sunspot Equilibria in Linear Rational Expectations Models. Journal of Economic Dynamics & Control 28, 273-285.

Lubik, Thomas and Frank Schorfheide, 2004. Testing for Indeterminacy: An Application to U.S. Monetary Policy. American Economic Review 94, 190-217.

Maliar, Lilia and John Taylor, 2019. Forward Guidance: Is It Useful Away From the Lower Bound?. NBER Working Paper 26053.

Mertens, Karel and Morten Ravn, 2014. Fiscal Policy in an Expectations-Driven Liquidity Trap. Review of Economic Studies 81, 1637-1667.

Nakata, Taisuke and Sebastian Schmidt, 2020. Expectations-Driven Liquidity Traps: Implications for Monetary and Fiscal Policy. CEPR Discussion Paper 15422.

Neusser, Klaus, 2019. Time-Varying Rational Expectations Models. Journal of Economic Dynamics & Control 107, 1-17.

Roeger, Werner, 2015. The Fiscal Multiplier and the Duration of the ZLB Constraint. Working Paper, European Commission.

APPENDICES

APPENDIX A) BASELINE MODEL WITH TIME-VARYING ZLB REGIMES (SECT. 3.2.3): DETERMINING THE DECISION RULE COEFFICIENTS

This Appendix shows how to solve for equilibrium decision rules in the baseline model of Sect. 3.2.3, using the method of undetermined coefficients. Sect. 3.2.3. assumes that the sunspot-driven ZLB regime follow a first-order Markov chain, and that the ZLB regime is independent across countries and independent of Home and Foreign fundamental forcing variables. The analysis focuses on equilibria, in which decision rules for (producer price) inflation are a function of the ZLB regime and of the natural real interest rate. In the baseline model (with a unitary trade elasticity), the two countries' Euler-Philipps equations are uncoupled (see Sect. 2.6). The equilibrium decision rule for country *k* inflation thus only depends on that country's ZLB regime and on its natural real interest rate (but not on the foreign ZLB regime or on the foreign natural rate). (See Appendix B for a detailed discussion of a model variant with a non-unitary trade elasticity; in that variant, a country's inflation decision depends on domestic *and* foreign ZLB regimes and natural rates.)

Denote country *k*=H,F inflation at date *t* by $\widehat{\Pi^B_{k,t}}$ when the country's ZLB constraint binds at date *t*, and by $\widehat{\Pi^S_{k,t}}$ when *k*'s ZLB constraint is slack at *t*. The decision rule for country *k* inflation has the following form:

$$\widehat{\Pi^B_{k,t}} = \mu^B + \lambda^B \widehat{r_{k,t}} \tag{19}$$

$$\text{and} \quad \widehat{\Pi^S_{k,t}} = \mu^S + \lambda^S \widehat{r_{k,t}}\ , \tag{20}$$

$$\text{with} \quad \gamma_\pi \widehat{\Pi^B_{k,t}} \leq -(\Pi - \beta)/\Pi < \gamma_\pi \widehat{\Pi^S_{k,t}}. \tag{21}$$

The first [second] inequality in (21) ensures that inflation $\widehat{\Pi^B_{k,t}}$ $[\widehat{\Pi^S_{k,t}}]$ is consistent with a binding [slack] ZLB constraint. (Recall that, in terms of hatted variables, the country k monetary policy rule is $\widehat{(1+i_{k,t+1})} = Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi_{k,t}}\}$; see (11).) Equilibrium decision rule coefficients $\mu^B, \lambda^B, \mu^S, \lambda^S$ are identical across countries (as countries are symmetric).

Denote country k's ZLB regime at date t by $z_{k,t} \in \{B,S\}$ where $z_{k,t}=B$ means that the ZLB constraint binds at date t (so that decision rule (19) applies) while $z_{k,t}=S$ indicates that the ZLB constraint is slack at t (so that decision rule (20) applies). Thus, $\widehat{\Pi_{k,t}}=\widehat{\Pi^B_{k,t}}$ when $z_{k,t}=B$ and $\widehat{\Pi_{k,t}}=\widehat{\Pi^S_{k,t}}$ when $z_{k,t}=S$.

Let the transition probabilities between country k ZLB regimes be denoted by $p_{mn}\equiv\text{Prob}(z_{k,t+1}=n|z_{k,t}=m)$ for $m,n \in \{B;S\}$, with $0\leq p_{mn}\leq 1$ and $p_{mB}+p_{mS}=1$. Transition probabilities are assumed to be identical across countries. Let $\Phi \equiv \begin{bmatrix} p_{BB} & p_{BS} \\ p_{SB} & p_{SS} \end{bmatrix}$ be the 2x2 matrix of transition probabilities between the ZLB regime at t and at t+1. Define $\widetilde{\Phi} \equiv \Phi\cdot\Phi = \begin{bmatrix} p_{BB}p_{BB}+p_{BS}p_{SB} & p_{BB}p_{BS}+p_{BS}p_{SS} \\ p_{SS}p_{SB}+p_{SB}p_{BB} & p_{SS}p_{SS}+p_{SB}p_{BS} \end{bmatrix}$. $\widetilde{\Phi}$ is the 2x2 matrix of transition probabilities between ZLB regimes at t and t+2: $\widetilde{\Phi_{1,1}}\equiv\text{Prob}(z_{k,t+2}=B|z_{k,t}=B)$, $\widetilde{\Phi_{1,2}}\equiv\text{Prob}(z_{k,t+2}=S|z_{k,t}=B)$, $\widetilde{\Phi_{2,1}}\equiv\text{Prob}(z_{k,t+2}=B|z_{k,t}=S)$, $\widetilde{\Phi_{2,2}}\equiv\text{Prob}(z_{k,t+2}=S|z_{k,t}=S)$, where $\widetilde{\Phi}_{i,j}$ denotes the element in the i-th row and j-th column of $\widetilde{\Phi}$.

Solving for decision-rule coefficients using method of undetermined coefficients

Equilibrium decision rule coefficients $\mu^B,\lambda^B,\mu^S,\lambda^S$ can be found using the method of undetermined coefficients, by substituting (19) and (20) into the Euler-Phillips equation (16) for country k=H,F:

$$Max\{-(\Pi-\beta)/\Pi,\gamma_\pi\cdot\widehat{\Pi_{k,t}}\}+\tfrac{1}{\kappa}\widehat{\Pi_{k,t}}=(1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi_{k,t+1}}-\tfrac{\beta}{\kappa}E_t\widehat{\Pi_{k,t+2}}+\widehat{r_{k,t}}\,. \tag{16}$$

When the decision rules (19),(20) hold, then conditional expected inflation in country k is a function of the country's ZLB regime $z_{k,t}$ and of the natural rate $\widehat{r_{k,t}}$: $E_t\widehat{\Pi_{k,t+s}}=E\{\widehat{\Pi_{k,t+s}}|z_{k,t},\widehat{r_{k,t}}\}$, for s≥0. This follows from the fact that (under the assumed stochastic processes for the ZLB regime and the natural rate), $z_{k,t}$ and $\widehat{r_{k,t}}$ are sufficient statistics for the conditional distribution of the future ZLB regime and of the future natural rate in country k. To determine $E\{\widehat{\Pi_{k,t+s}}|z_{k,t},\widehat{r_{k,t}}\}$, we can use the fact that, by the law of iterated expectations,

$E\{\widehat{\Pi_{k,t+1}}|z_{k,t},\widehat{r_{k,t}}\} = E(E\{\widehat{\Pi_{k,t+1}}|z_{k,t+1},\widehat{r_{k,t}}\} \mid z_{k,t},\widehat{r_{k,t}})$.

If country k is in ZLB regime 'B' at date $t+1$, i.e. $z_{k,t+1}=B$, then $\widehat{\Pi_{k,t+1}}=\mu^B+\lambda^B\widehat{r_{k,t+1}}$. Thus, $E\{\widehat{\Pi_{k,t+1}}|z_{k,t+1}=B,\widehat{r_{k,t}}\}=\mu^B+\lambda^B\rho\widehat{r_{k,t}}$ because $E\{\widehat{r_{k,t+1}}|z_{t+1}=B,\widehat{r_{k,t}}\}=E\{\widehat{r_{k,t+1}}|\widehat{r_{k,t}}\}=\rho\widehat{r_{k,t}}$ (as the natural rate and the ZLB regime are assumed to be independent random variables; the natural rate follows an AR(1) process with autocorrelation ρ). By the same logic $E\{\widehat{\Pi_{k,t+1}}|z_{t+1}=S,\widehat{r_{k,t}}\}=\mu^S+\lambda^S\rho\widehat{r_{k,t}}$.

If country k is in ZLB regime 'B' at date t, then the country's ZLB regime at t+1 will be $z_{k,t+1}=B$ with probability p_{BB}, and $z_{k,t+1}=S$ with probability p_{BS}. Conditional on $z_{k,t}=B$ and $\widehat{r_{k,t}}$, the expected country k inflation rate at t+1 is, thus:

$$E\{\widehat{\Pi_{k,t+1}}|z_{k,t}=B,\widehat{r_{k,t}}\} = p_{BB}E\{\widehat{\Pi_{k,t+1}}|z_{k,t+1}=B,\widehat{r_{k,t}}\}+ p_{BS}E\{\widehat{\Pi_{k,t+1}}|z_{k,t+1}=S,\widehat{r_{k,t}}\}.$$

Hence, $E\{\widehat{\Pi_{k,t+1}}|z_{k,t}=B,\widehat{r_{k,t}}\} = p_{BB}\{\mu^B+\lambda^B\rho\widehat{r_{k,t}}\}+ p_{BS}\{\mu^S+\lambda^S\rho\widehat{r_{k,t}}\}$, which can be expressed as

$$E\{\widehat{\Pi_{k,t+1}}|z_{k,t}=B,\widehat{r_{k,t}}\} = [p_{BB}\mu^B+p_{BS}\mu^S]+[p_{BB}\lambda^B+p_{BS}\lambda^S]\rho\widehat{r_{k,t}}. \tag{A.1}$$

Let $\mu\equiv[\mu^B;\mu^S]$ and $\lambda\equiv[\lambda^B;\lambda^S]$ denote 2x1 column vectors that, respectively, include the intercepts and the slopes of the inflation decision rules in the two ZLB regimes. Note that $[p_{BB}\mu^B+p_{BS}\mu^S]=\Phi_{1,\bullet}\times\mu$ and $[p_{BB}\lambda^B+p_{BS}\lambda^S]=\Phi_{1,\bullet}\times\lambda$, where $\Phi_{1,\bullet}=[p_{BB}\ \ p_{BS}]$ is a 1x2 row vector that corresponds to the first row of 'one-period ahead' ZLB-regime transition probability matrix Φ. (A.1) thus gives the following formula for expected inflation at date t+1, conditional on a binding ZLB constraint at t $(z_{k,t}=B)$ and on $\widehat{r_{k,t}}$:

$$E(\widehat{\Pi_{k,t+1}}|z_{k,t}=B,\widehat{r_{k,t}})=\Phi_{1,\bullet}\{\mu+\lambda\rho\widehat{r_{k,t}}\}. \tag{A.2}$$

Analogously, conditional expected date t+1 inflation, given a slack ZLB constraint at date t $(z_{k,t}=S)$ and $\widehat{r_{k,t}}$ is: $E\{\widehat{\Pi_{k,t+1}}|z_{k,t}=S,\widehat{r_{k,t}}\} = p_{SB}\{\mu^B+\lambda^B\rho\widehat{r_{k,t}}\}+ p_{SS}E\{\mu^S+\lambda^S\rho\widehat{r_{k,t}}\}$, which can be written as:

$$E(\widehat{\Pi_{k,t+1}}|z_{k,t}=S,\widehat{r_{k,t}})=\Phi_{2,\bullet}\{\mu+\lambda\rho\widehat{r_{k,t}}\}, \tag{A.3}$$

where $\Phi_{2,\bullet}=[p_{SB}\ \ p_{SS}]$ is the second row of the ZLB-regime transition probability matrix Φ.

Similar logic shows that 'two-periods ahead' expectations of country k inflation, conditional on, respectively, a binding and slack ZLB regime at date t are

$$E(\widehat{\Pi_{k,t+2}}|z_{k,t}=B,\widehat{r_{k,t}})=\widetilde{\Phi_{1,\bullet}}\{\mu+\lambda\rho^2\widehat{r_{k,t}}\} \quad \text{and} \quad E(\widehat{\Pi_{k,t+2}}|z_{k,t}=S,\widehat{r_{k,t}})=\widetilde{\Phi_{2,\bullet}}\{\mu+\lambda\rho^2\widehat{r_{k,t}}\}, \qquad \text{(A.4)}$$

where $\widetilde{\Phi_{1,\bullet}}$ and $\widetilde{\Phi_{2,\bullet}}$ are 1x2 row vectors corresponding, respectively, to the first and second rows of the 'two-periods ahead' transition probability matrix $\widetilde{\Phi}$. Equations (A.2)-(A.4) are stated as (24) in the main text (see Sect. 3.2.3).

(A.2)-(A.4) show that conditional expected inflation depends on the decision rule coefficients μ and λ. This allows to determine μ,λ using the method of undetermined coefficients, from the Euler-Phillips equation (16).

When the country k ZLB constraint binds at date t, so that $z_{k,t}=B$ and $\widehat{\Pi_{k,t}}=\widehat{\Pi_{k,t}^B}\leq-(1/\gamma_\pi)(\Pi-\beta)/\Pi$ (see (21)), the date t Euler-Phillips equation for country k is $-(\Pi-\beta)/\Pi=-\frac{1}{\kappa}\widehat{\Pi_{k,t}^B}+(1+\frac{1+\beta}{\kappa})E\{\widehat{\Pi_{k,t+1}}|z_{k,t}=B,\widehat{r_{k,t}}\}-\frac{\beta}{\kappa}E\{\widehat{\Pi_{k,t+2}}|z_{k,t}=B,\widehat{r_{k,t}}\}+\widehat{r_{k,t}}$. Using (19), (A.2) and (A.4), this implies equation (25) in the main text:

$$-(\Pi-\beta)/\Pi=-\tfrac{1}{\kappa}\{\mu^B+\lambda^B\widehat{r_{k,t}}\}+(1+\tfrac{1+\beta}{\kappa})\Phi_{1,\bullet}\{\mu+\lambda\rho\widehat{r_{k,t}}\}-\tfrac{\beta}{\kappa}\widetilde{\Phi}_{1,\bullet}\{\mu+\lambda\rho^2\widehat{r_{k,t}}\}+\widehat{r_{k,t}}\,. \qquad (25)$$

When the country k ZLB constraint is slack at date t, i.e. $z_{k,t}=S$ and $\widehat{\Pi_{k,t}}=\widehat{\Pi_{k,t}^S}>-(1/\gamma_\pi)(\Pi-\beta)/\Pi$, then the date t Euler-Phillips equation for country k is $\gamma_\pi\widehat{\Pi_{k,t}^S}=-\frac{1}{\kappa}\widehat{\Pi_{k,t}^S}+(1+\frac{1+\beta}{\kappa})E\{\widehat{\Pi_{k,t+1}}|z_{k,t}=S,\widehat{r_{k,t}}\}-\frac{\beta}{\kappa}E\{\widehat{\Pi_{k,t+2}}|z_{k,t}=S,\widehat{r_{k,t}}\}+\widehat{r_{k,t}}$. Using (20), (A.3) and (A.4), this gives equation (26) in the main text:

$$\gamma_\pi\{\mu^S+\lambda^S\widehat{r_{k,t}}\}=-\tfrac{1}{\kappa}\{\mu^S+\lambda^S\widehat{r_{k,t}}\}+(1+\tfrac{1+\beta}{\kappa})\Phi_{2,\bullet}\{\mu+\lambda\rho\widehat{r_{k,t}}\}-\tfrac{\beta}{\kappa}\widetilde{\Phi}_{2,\bullet}\{\mu+\lambda\rho^2\widehat{r_{k,t}}\}+\widehat{r_{k,t}}\,. \qquad (26)$$

The Euler-Phillips equation (25) holds for arbitrary values of the real natural interest rate $\widehat{r_{k,t}}$ iff

$$-(\Pi-\beta)/\Pi=-\tfrac{1}{\kappa}\mu^B+(1+\tfrac{1+\beta}{\kappa})\Phi_{1,\bullet}\mu-\tfrac{\beta}{\kappa}\widetilde{\Phi}_{1,\bullet}\mu \qquad \text{(A.5)}$$

$$\text{and} \quad 0=-\tfrac{1}{\kappa}\lambda^B+(1+\tfrac{1+\beta}{\kappa})\Phi_{1,\bullet}\lambda\rho-\tfrac{\beta}{\kappa}\widetilde{\Phi}_{1,\bullet}\lambda\rho^2+1\,. \qquad \text{(A.6)}$$

Euler-Phillips equation (26) holds for arbitrary $\widehat{r_{k,t}}$ iff

$$0=-(\tfrac{1}{\kappa}+\gamma_\pi)\mu^S+(1+\tfrac{1+\beta}{\kappa})\Phi_{2,\bullet}\mu-\tfrac{\beta}{\kappa}\widetilde{\Phi}_{2,\bullet}\mu \qquad \text{(A.7)}$$

$$\text{and} \quad 0=-(\tfrac{1}{\kappa}+\gamma_\pi)\lambda^S+(1+\tfrac{1+\beta}{\kappa})\Phi_{2,\bullet}\lambda\rho-\tfrac{\beta}{\kappa}\widetilde{\Phi}_{2,\bullet}\lambda\rho^2+1\,. \qquad \text{(A.8)}$$

(A.5)-(A.8) is a system of four equations in the four decision rule coefficients $\mu^B, \mu^S, \lambda^B, \lambda^S$ (recall that $\mu \equiv [\mu^B; \mu^S]$ and $\lambda \equiv [\lambda^B; \lambda^S]$).

Stacking (A.5) and (A.7) gives

$$\begin{bmatrix} -(\Pi-\beta)/\Pi \\ 0 \end{bmatrix} = \left\{ \begin{bmatrix} -\frac{1}{\kappa} & 0 \\ 0 & -\gamma_\pi - \frac{1}{\kappa} \end{bmatrix} + (1+\tfrac{1+\beta}{\kappa})\Phi - \tfrac{\beta}{\kappa}\widetilde{\Phi} \right\} \cdot \mu, \tag{A.9}$$

while stacking (A.6) and (A.8) gives

$$\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \left\{ \begin{bmatrix} -\frac{1}{\kappa} & 0 \\ 0 & -\gamma_\pi - \frac{1}{\kappa} \end{bmatrix} + (1+\tfrac{1+\beta}{\kappa})\Phi\rho - \tfrac{\beta}{\kappa}\widetilde{\Phi}\rho^2 \right\} \cdot \lambda, \tag{A.10}$$

where the fact was used that $\Phi = \begin{bmatrix} \Phi_{1,\bullet} \\ \Phi_{2,\bullet} \end{bmatrix}$ and $\widetilde{\Phi} = \begin{bmatrix} \widetilde{\Phi_{1,\bullet}} \\ \widetilde{\Phi_{2,\bullet}} \end{bmatrix}$. Solving (A.9) and (A.10) for μ and λ gives equation (28) in the main text:

$$\mu = \left\{ \begin{bmatrix} -\frac{1}{\kappa} & 0 \\ 0 & -\gamma_\pi - \frac{1}{\kappa} \end{bmatrix} + (1+\tfrac{1+\beta}{\kappa})\Phi - \tfrac{\beta}{\kappa}\widetilde{\Phi} \right\}^{-1} \begin{bmatrix} -(\Pi-\beta)/\Pi \\ 0 \end{bmatrix}$$

$$\text{and } \lambda = \left\{ \begin{bmatrix} -\frac{1}{\kappa} & 0 \\ 0 & -\gamma_\pi - \frac{1}{\kappa} \end{bmatrix} + (1+\tfrac{1+\beta}{\kappa})\Phi\rho - \tfrac{\beta}{\kappa}\widetilde{\Phi}\rho^2 \right\}^{-1} \begin{bmatrix} -1 \\ -1 \end{bmatrix}. \tag{28}$$

The following condition ensures that the inequality constraints (21) hold for values of $\widehat{r_{k,t}}$ sufficiently close to zero:

$$\gamma_\pi \mu^B < -(\Pi-\beta)/\Pi < \gamma_\pi \mu^S. \tag{29}$$

As discussed in Sect. 3.2.3, the existence of an equilibrium with sunspot-driven ZLB regimes that follow a Markov chain requires probabilities p_{SS} and p_{BB} that are close to unity; when p_{SS} and p_{BB} are *not* sufficiently close to unity, then the vector $\mu=[\mu^B;\mu^S]$ determined by (A.9) violates the inequalities (29).

APPENDIX B) MODEL VERSION WITH NON-UNITARY TRADE ELASTICITY

This Appendix discusses a model variant in which the Cobb-Douglas Home consumption aggregator (see Sect. 2.1 in the main text) is replaced by the CES aggregator: $C_{H,t}\equiv\{\xi^{1/\phi}(Y_{H,t}^{H})^{(\phi-1)/\phi}+(1-\xi)^{1/\phi}(Y_{H,t}^{F})^{(\phi-1)/\phi}\}^{\phi/(\phi-1)}$ where ϕ (with ϕ>0, $\phi\neq1$) is the substitution elasticity between (aggregate) domestic and imported intermediates $(Y_{H,t}^{H},Y_{H,t}^{F})$. The Cobb-Douglas aggregator implies a unit substitution elasticity, ϕ=1.[29] As before, $\frac{1}{2}<\xi<1$ is assumed (consumption home bias). The demand for domestic and imported intermediates by the Home consumer is now given by $Y_{H,t}^{H}=\xi\cdot C_{H,t}\cdot(P_{H,t}/CPI_{H,t})^{-\phi}$ and $Y_{H,t}^{F}=(1-\xi)\cdot C_{H,t}\cdot([P_{F,t}/S_t]/CPI_{H,t})^{-\phi}$, where $CPI_{H,t}\equiv[\xi\cdot(P_{H,t})^{1-\phi}+(1-\xi)\cdot(P_{F,t}/S_t)^{1-\phi}]^{1/(1-\phi)}$ is the country H final consumption price (i.e. the marginal cost of the final consumption good).

The Home terms of trade and the real exchange rate are defined as $q_t\equiv S_tP_{H,t}/P_{F,t}$ and $RER_t\equiv S_tCPI_{H,t}/CPI_{F,t}$, respectively. Note that $RER_t=\{[\xi\cdot(q_t)^{1-\phi}+(1-\xi)]/[\xi+(1-\xi)\cdot(q_t)^{1-\phi}]\}^{1/(1-\phi)}$. Due to household consumption home bias (2ξ-1>0), the real exchange rate is an increasing function of the terms of trade. The real price of the Home domestic intermediate good, in units of Home final consumption, $P_{H,t}/CPI_{H,t}=q_t/[\xi\cdot(q_t)^{1-\phi}+(1-\xi)]^{1/(1-\phi)}$, too is an increasing function of the terms of trade. Linearization of these equations around a symmetric deterministic steady state gives: $\widehat{RER}_t=(2\xi-1)\cdot\widehat{q}_t$ and $\widehat{P_{H,t}/CPI_{H,t}}=(1-\xi)\cdot\widehat{q}_t$. The real price of Foreign intermediates (in units of Foreign final consumption) obeys $\widehat{P_{F,t}/CPI_{F,t}}=-(1-\xi)\cdot\widehat{q}_t$.

Using the above intermediate good demand functions, the market clearing conditions for Home and Foreign intermediates can be expressed as

$$Y_{H,t}=\xi\cdot C_{H,t}\cdot(P_{H,t}/CPI_{H,t})^{-\phi}+(1-\xi)\cdot C_{F,t}\cdot([P_{H,t}S_t]/CPI_{F,t})^{-\phi}+G_{H,t}$$

and $$Y_{F,t}=(1-\xi)\cdot C_{H,t}\cdot([P_{F,t}/S_t]/CPI_{H,t})^{-\phi}+\xi C_{F,t}\cdot(P_{F,t}/CPI_{F,t})^{-\phi}+G_{F,t}.$$

Linearization of these market clearing conditions around a symmetric steady state with zero government purchases gives:

$$\widehat{Y_{H,t}}=\xi\widehat{C_{H,t}}+(1-\xi)\widehat{C_{F,t}}-2\phi\xi(1-\xi)\widehat{q}_t+\widehat{G_{H,t}} \quad\text{and}\quad \widehat{Y_{F,t}}=(1-\xi)\widehat{C_{H,t}}+\xi\widehat{C_{F,t}}+2\xi(1-\xi)\phi\widehat{q}_t+\widehat{G_{F,t}}. \quad \text{(B.1)}$$

[29] The Cobb-Douglas aggregator is the limit of the CES aggregator as $\phi\rightarrow1$.

The market clearing conditions (7) in the baseline model are a special case (for ϕ=1) of (B.1). The other linearized aggregate equilibrium conditions do not involve the trade elasticity ϕ, and thus these conditions continue to hold unchanged when $\phi \neq 1$ is assumed, namely the risk sharing condition (6), the Phillips equations (9), the equations defining real marginal cost (10), the Euler equations (8) and the monetary policy interest rate rules (11). These equations are restated here, for convenience:

$$\widehat{C_{H,t}} - \widehat{C_{F,t}} = -(2\xi - 1)\widehat{q_t} + \widehat{\Psi_{H,t}} - \widehat{\Psi_{F,t}}\,; \tag{B.2}$$

$$\widehat{\Pi_{k,t}} = \kappa_w \cdot \widehat{mc_{k,t}} + \beta E_t \widehat{\Pi_{k,t+1}} \quad \text{for } k=\text{H,F}; \tag{B.3}$$

$$\widehat{mc_{H,t}} = \widehat{C_{H,t}} + \tfrac{1}{\eta}\widehat{Y_{H,t}} - (1+\tfrac{1}{\eta})\widehat{\theta_{H,t}} - (1-\xi)\widehat{q_t} \quad \text{and} \quad \widehat{mc_{F,t}} = \widehat{C_{F,t}} + \tfrac{1}{\eta}\widehat{Y_{F,t}} - (1+\tfrac{1}{\eta})\widehat{\theta_{F,t}} + (1-\xi)\widehat{q_t}; \tag{B.4}$$

$$\widehat{1+i_{k,t+1}} = E_t\{\widehat{\Pi^{CPI}_{k,t+1}} + \widehat{C_{k,t+1}} - \widehat{C_{k,t}} + \widehat{\Psi_{k,t}} - \widehat{\Psi_{k,t+1}}\} \quad \text{for } k=\text{H,F}; \tag{B.5}$$

$$(\widehat{1+i_{k,t+1}}) = Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi_{k,t}}\} \quad \text{for } k=\text{H,F}. \tag{B.6}$$

The model can be solved in the following steps: (I) Use the static model equations (B.1),(B.2),(B.4) and the Phillips equations (B.3) to express Home and Foreign output, consumption and the terms of trade as functions of current and expected inflation and of the exogenous variables. (II) Substitute the resulting formulae for consumption and the terms of trade into the Euler equations, to write the Euler equations in terms of inflation and exogenous variables. (III) Find an inflation process that satisfies those Euler equations.

Let $A_t \equiv (\widehat{Y_{H,t}}, \widehat{Y_{F,t}}, \widehat{C_{H,t}}, \widehat{C_{F,t}}, \widehat{q_t})$ be a 5x1 column vector containing Home and Foreign output, consumption and the terms of trade. Let $B_t \equiv (\widehat{\Pi_{H,t}}, \widehat{\Pi_{F,t}}, E_t\widehat{\Pi_{H,t+1}}, E_t\widehat{\Pi_{F,t+1}})$ be a column vector of current and expected future Home and Foreign producer price inflation, and let $X_t \equiv (\widehat{\theta_{H,t}}, \widehat{\theta_{F,t}}, \widehat{G_{H,t}}, \widehat{G_{F,t}}, \widehat{\Psi_{H,t}}, \widehat{\Psi_{F,t}})$ be a column vector listing the exogenous variables.

(B.1)-(B.4) defines a system of 5 equations in A_t, B_t, X_t that can be used to express the vector A_t as linear functions of B_t and X_t :

$$A_t = \Gamma_1 B_t + \Gamma_2 X_t, \tag{B.7}$$

where Γ_1 and Γ_2 are 7x4 and 7x6 matrices, respectively, whose elements are functions of the model parameters.

Let $H \equiv \frac{1+\eta}{\eta} > 1$, $\Xi \equiv 2\xi(1-\xi) > 0$ and $D \equiv \{(\phi-1)2(H-1)\Xi + H\} > 0$. Country k output, consumption, net exports and the terms of trade can be expressed as functions of current and expected future inflation and of exogenous shocks:

$$\widehat{Y_{k,t}} = \tfrac{1}{\kappa D}\{(\phi-1)(2H-1)\Xi+H\}\cdot[\widehat{\Pi_{k,t}} - \beta E_t \widehat{\Pi_{k,t+1}}] - \tfrac{1}{\kappa D}(\phi-1)\Xi\cdot[\widehat{\Pi_{l,t}} - \beta E_t \widehat{\Pi_{l,t+1}}] +$$
$$\tfrac{1}{D}\{(\phi-1)(2H-1)\Xi+H\}\cdot\widehat{\theta_{k,t}} - \tfrac{1}{D}(\phi-1)\Xi\cdot\widehat{\theta_{l,t}} +$$
$$\tfrac{1}{D}\{(\phi-1)\tfrac{H-1}{H}\Xi + 1\}\cdot\widehat{G_{k,t}} + \tfrac{1}{D}\tfrac{H-1}{H}\Xi(\phi-1)\cdot\widehat{G_{l,t}} -$$
$$\tfrac{1}{D}((\phi-1)\Xi + 1-\xi)\cdot(\widehat{\Psi_{kt}} - \widehat{\Psi_{lt}}), \quad \text{for } k,l \in \{\text{H,F}\},\ l \neq \text{k}, \qquad \text{(B.8)}$$

$$\widehat{C_{k,t}} = \tfrac{1}{\kappa D}\{(\phi-1)(H-1)\Xi+\xi H\}\cdot[\widehat{\Pi_{k,t}} - \beta E_t\widehat{\Pi_{k,t+1}}] + \tfrac{1}{\kappa D}\{(\phi-1)(H-1)\Xi+(1-\xi)H\}\cdot[\widehat{\Pi_{l,t}} - \beta E_t\widehat{\Pi_{l,t+1}}] +$$
$$\tfrac{1}{D}\{(\phi-1)(H-1)\Xi+\xi H\}\cdot\widehat{\theta_{k,t}} + \tfrac{1}{D}\{(\phi-1)(H-1)\Xi+(1-\xi)H\}\cdot\widehat{\theta_{l,t}} -$$
$$\tfrac{1}{D}(H-1)\{(\phi-1)\Xi\tfrac{H-1}{H}+\xi\}\cdot\widehat{G_{k,t}} - \tfrac{1}{D}(H-1)\{(\phi-1)\Xi\tfrac{H-1}{H}+1-\xi\}\cdot\widehat{G_{l,t}} +$$
$$\tfrac{1}{D}\cdot\{\phi(H-1)\Xi+1-\xi\}\cdot(\widehat{\Psi_{kt}} - \widehat{\Psi_{lt}}) \text{ for } k,l \in \{\text{H,F}\},\ k \neq l; \qquad \text{(B.9)}$$

$$NX_{k,t} = \tfrac{1}{\kappa D}H\Xi(\phi-1)\cdot[(\widehat{\Pi_{k,t}} - \beta E_t\widehat{\Pi_{k,t+1}}) - (\widehat{\Pi_{l,t}} - \beta E_t\widehat{\Pi_{l,t+1}})] +$$
$$\tfrac{1}{D}H\Xi(\phi-1)\cdot(\widehat{\theta_{k,t}} - \widehat{\theta_{l,t}}) - \tfrac{1}{D}(H-1)\Xi(\phi-1)\cdot(\widehat{G_{k,t}} - \widehat{G_{l,t}}) -$$
$$\tfrac{1}{D}H(1-\xi)\{(\phi-1)2\xi+1\}\cdot(\widehat{\Psi_{k,t}} - \widehat{\Psi_{l,t}}) \quad \text{for } k,l \in \{\text{H,F}\},\ l \neq k; \qquad \text{(B.10)}$$

$$\widehat{q_t} = -\tfrac{1}{\kappa D}H[(\widehat{\Pi_{H,t}} - \widehat{\Pi_{F,t}}) - \beta(E_t\widehat{\Pi_{H,t+1}} - E_t\widehat{\Pi_{F,t+1}})] -$$
$$\tfrac{1}{D}H(\widehat{\theta_{H,t}} - \widehat{\theta_{F,t}}) + \tfrac{1}{D}(H-1)(\widehat{G_{H,t}} - \widehat{G_{F,t}}) + \tfrac{1}{D}\{(2\xi-1)(H-1)+1\}(\widehat{\Psi_{H,t}} - \widehat{\Psi_{F,t}}); \qquad \text{(B.11)}$$

These equations hold both at the ZLB and away from the ZLB. Note that, for a unitary trade elasticity, ϕ=1, the slope coefficients of foreign inflation, foreign productivity and foreign government purchases in the country k output equation are zero: output just depends on domestic inflation, domestic productivity, domestic government purchases and domestic and foreign preference shocks. When ϕ=1, then net exports only depend on (domestic and foreign) preference shocks. By contrast, for $\phi\neq1$, domestic *and* foreign inflation and productivity and government purchases shocks affect output and net exports.

To complete the model solution, we substitute the preceding equations into the Home and Foreign Euler equations (B.5). A country's Euler equation involves the growth rate of nominal consumption spending (in national currency); see (B.5). That growth rate can be written as:

$$\widehat{\Pi^{CPI}_{k,t+1}}+\widehat{C_{k,t+1}}-\widehat{C_{k,t}}=\widehat{\Pi_{k,t+1}}+\widehat{Z_{k,t+1}}-\widehat{Z_{k,t}}\,,\quad \text{for } k=\text{H,F}, \tag{B.12}$$

with $Z_{H,t}\equiv\widehat{C_{H,t}}-(1-\xi)\widehat{q_t}$ and $Z_{F,t}\equiv\widehat{C_{F,t}}+(1-\xi)\widehat{q_t}$ (see Sect. 2.6). Using (B.9) and (B.11), we can express $Z_{k,t}$ as

$$Z_{k,t}=\tfrac{1}{\kappa D}\{(\phi-1)(H-1)\Xi+H\}\cdot[\widehat{\Pi_{k,t}}-\beta E_t\widehat{\Pi_{k,t+1}}]+\tfrac{1}{\kappa D}(\phi-1)(H-1)\Xi\cdot[\widehat{\Pi_{l,t}}-\beta E_t\widehat{\Pi_{l,t+1}}]+Z^{Flex}_{k,t}, \tag{B.13}$$

$$\text{with } Z^{Flex}_{k,t}\equiv\tfrac{1}{D}\{(\phi-1)(H-1)\Xi+H\}\cdot\widehat{\theta_{k,t}}+\tfrac{1}{D}(\phi-1)(H-1)\Xi\cdot\widehat{\theta_{l,t}}-$$

$$\tfrac{1}{D}(H-1)\{(\phi-1)\tfrac{H-1}{H}\Xi+1\}\cdot\widehat{G_{k,t}}-\tfrac{1}{D}(\phi-1)\tfrac{(H-1)^2}{H}\Xi\cdot\widehat{G_{l,t}}+$$

$$\tfrac{1}{D}(H-1)\{\Xi(\phi-1)+1-\xi\}\cdot(\widehat{\Psi_{kt}}-\widehat{\Psi_{lt}})\quad \text{for } k,l\in\{\text{H,F}\},\ l\neq k. \tag{B.14}$$

Write (B.13) as

$$Z_{k,t}=\Gamma_k B_t+Z^{Flex}_{k,t}\quad \text{for k=H,F}, \tag{B.15}$$

where Γ_k is a 1x4 row vector. $Z^{Flex}_{k,t}$ is a function only of exogenous variables. In a flex-prices world the slope of the Phillips curve is infinite: $\kappa=\infty$. Thus, under flexible prices, the slope coefficients of inflation in (B.13) are zero, so that then $Z_{k,t}=Z^{Flex}_{k,t}$.

Using (B.12) and (B.15), the Euler equations (B.5) can be expressed in terms of the nominal interest rate, inflation and exogenous variables:

$$\widehat{1+i_{k,t+1}}=E_t\{\widehat{\Pi_{k,t+1}}+\Gamma_k(B_{t+1}-B_t)\}+\widehat{r_{k,t}}\quad \text{for k=H,F} \tag{B.16}$$

$$\text{with } \widehat{r_{k,t}}\equiv(1-\rho)\{\widehat{\Psi_{k,t}}-Z^{Flex}_{k,t}\}. \tag{B.17}$$

where I used the fact that $E_t\widehat{\Psi_{k,t+1}}=\rho\widehat{\Psi_{k,t}}$ and $E_tZ^{Flex}_{k,t+1}=\rho Z^{Flex}_{k,t}$ (as all forcing variables follow univariate AR(1) process with autocorrelation ρ).

$\widehat{r_{k,t}}$ is the country k expected gross real interest rates (expressed as a relative deviation from the steady state gross real rate), defined in units of country k output, that would obtain in a flex-prices world. (Note that $\widehat{r_{k,t}}=\widehat{1+i_{k,t+1}}-E_t\widehat{\Pi_{k,t+1}}$ holds in a flex-prices world, as there $\Gamma_k=0$). I refer to $\widehat{r_{k,t}}$ as country k's natural real interest rate (see Sect. 2.6). $\widehat{r_{k,t}}$ is a function of only exogenous variables.

Let $D_t \equiv (\widehat{\Pi_{H,t}}, \widehat{\Pi_{F,t}}, E_t\widehat{\Pi_{H,t+1}}, E_t\widehat{\Pi_{F,t+1}}, E_t\widehat{\Pi_{H,t+2}}, E_t\widehat{\Pi_{F,t+2}})$ be the 6x1 column vector containing Home and Foreign inflation at date *t* and expected inflation at *t*+1 and *t*+2.

Combining the monetary policy rule (B.6) with Euler equation (B.16) gives:

$$Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi_{k,t}}\}=\Lambda_k D_t + r_{k,t}\text{, for k=H,F,} \qquad \text{(B.18)}$$

where Λ_k is a 1x6 row vector of coefficients. I refer to this equation as the "Euler-Phillips" equation (see Sect. 2.6).

To solve the model, we have to find processes for Home and Foreign inflation that solve the Euler-Phillips equation (B.18) for k=H,F. Once such processes have been determined, output, consumption, net exports and the terms of trade can be determined using (B.8)-(B.11).

Under a unitary trade elasticity, ϕ=1, the two countries' Euler-Phillips equations are uncoupled: country k's Euler equation depends on domestic inflation, but not on foreign inflation; this follows from the fact that, for ϕ=1, $Z_{k,t}$ does not depend on foreign inflation (see (B.13)). Also, for ϕ=1, $Z_{k,t}^{Flex}$ and the natural real interest rate do not depend on foreign productivity and government purchases. As mentioned above, country k output does not depend on foreign inflation, productivity and government purchases when ϕ=1 (see (B.8)), as discussed in the main text.

By contrast, for $\phi\neq1$, the country k Euler-Phillips equation depends on domestic and foreign inflation, and the country's natural real interest rate depends on domestic and foreign productivity and government purchases. When $\phi>1$ (as assumed in Sect. 5), then the natural real interest rate is decreasing in domestic and foreign productivity, and increasing in domestic and foreign government purchases; however, the natural rate depends more strongly on domestic forcing variables than on foreign forcing variables. [30]

As in the baseline model with ϕ=1 discussed in the main text, there are multiple steady states when $\phi \neq 1$. In steady state, the country k Euler-Phillips equation is $Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi_k}\} = \widehat{\Pi_k}$ (from (B.16)). Given our assumption that the Taylor principle holds (γ_π>1), this equation is solved by two steady state inflation rates: $\widehat{\Pi_k}=0$ and $\widehat{\Pi_k}=-(\Pi-\beta)/\Pi$. The ZLB binds in the latter steady state. Note that, in steady state, the country k Euler-Phillips equation only depends on country k inflation. In steady state, the two countries' Euler-Phillips equations are, thus, uncoupled. A steady state liquidity trap can arise in country H, irrespective of whether there is a liquidity trap in country Foreign, and vice versa.

Expectations-driven liquidity traps

I construct equilibria with expectations-driven liquidity traps by assuming random self-fulfilling switches in agents' inflation expectations. In the equilibria studied here, inflation in each country is a function of both countries' ZLB regimes and of their natural real interest rates. Under a unitary trade elasticity ϕ=1 (as assumed in Sect. 3), the two countries' Euler-Phillips equations are uncoupled, and a country's equilibrium inflation decision rule only depends on the domestic ZLB regime and on the domestic natural real interest rate. However, for ϕ>1, the two countries' Euler-Phillips equations are linked, and thus equilibrium inflation decision rules depend on the domestic *and* foreign ZLB regimes and on domestic *and* foreign natural real interest rates. Assume that the ZLB regimes follow a Markov chain. Denote the ZLB regime as $z_t \in \{BB, BS, SB, SS\}$ where $z_t = hf$ indicates that the Home ZLB state is h$\in$\{B,S\} while the Foreign ZLB state is f$\in$\{B,S\} at date *t*; "B" indicates that the ZLB binds (liquidity trap), while "S" indicates that the ZLB constraint is slack. E.g. $z_t = BS$ indicates that, at date *t*, the Home ZLB constraint binds, while the Foreign ZLB constraint is slack. Let $\widehat{\Pi_{k,t}^{hf}}$ denote the country *k*

[30] This follows from the fact that, in the equation for $Z_{k,t}$, the slope coefficients of domestic productivity and government purchases are greater (in absolute values) than the coefficients of foreign productivity and government purchases, respectively (see (A.13), (A.14)).

inflation rate at date t, when the Home ZLB regime is h, while the Foreign ZLB regime is f, with h,f$\in\{B,S\}$. Home and Foreign inflation decision rules are given by:

$$\widehat{\Pi_{k,t}^{hf}} = \mu_k^{hf} + \lambda_k^{hf}\,\widehat{r_{H,t}} + \zeta_k^{hf}\,\widehat{r_{F,t}}\,, \tag{B.19}$$

$$\text{with} \quad \gamma_\pi \widehat{\Pi_{H,t}^{Bf}} \le -(\Pi-\beta)/\Pi < \gamma_\pi \widehat{\Pi_{H,t}^{Sf}}$$

$$\text{and} \quad \gamma_\pi \widehat{\Pi_{F,t}^{hB}} \le -(\Pi-\beta)/\Pi < \gamma_\pi \widehat{\Pi_{F,t}^{hS}}$$

for country k$\in\{H,F\}$ and Home/Foreign ZLB regimes h,f$\in\{B,S\}$.

The numerical simulations assume that the two countries' ZLB regimes are independent (see Sect. 3.2.3).

The coefficients of the Home and Foreign decision rules can be determined using the method of undetermined coefficients, after substituting (B.19) into the Euler-Phillips equation (B.18). For the trade elasticity ϕ=1.5 assumed in the model simulations discussed in Sect. 5 it is again (as in the ϕ=1 case discussed in Sect. 3) found that the existence of an equilibrium with an occasionally binding ZLB constraint requires persistent ZLB regimes, i.e. the probabilities p_{SS} and p_{BB} defining the persistence of the ZLB regimes (see Sect. 3.2.3) have to be close to unity. The numerical simulations of model variants with occasionally binding ZLB constraints assume $p_{BB}=p_{SS}=0.95$. (Note: as the two countries are symmetric, the equilibrium decision rules are symmetric, i.e. $\mu_H^{BB}=\mu_F^{BB}$, $\lambda_H^{BB}=\zeta_F^{BB}$, $\lambda_F^{BB}=\zeta_H^{BB}$ hold etc.)

Fundamentals-driven liquidity traps

With a non-unitary trade elasticity, $\phi\neq 1$, computation of the fundamentals-driven liquidity trap proceeds along the same lines as in the ϕ=1 case discussed in Sect. 4. As in Sect. 4 it is assumed that fundaments-driven liquidity traps are brought about by unanticipated one-time preference shocks (Ψ) at some date t=0 that depress natural real interest rates. The simulations of fundaments-driven liquidity traps presented here assume that the economy evolves deterministically (perfect foresight), after t=0. As there are no exogenous innovations after date t=0, the natural real interest rate in country k=H,F at $t\geq 0$ is: $\widehat{r_{k,t}}=\rho^t\cdot\widehat{r_{k,0}}$, where $0<\rho<1$ is the autocorrelation of the exogenous forcing processes (and of the natural rate).

A key difference compared to the ϕ=1 case is that, with $\phi \neq 1$, country k's unconstrained inflation rate and the unconstrained nominal interest rate (i.e. the inflation and interest rates that would obtain in a world without ZLB constraints) depend on domestic and foreign natural real interest rates. E.g., the Home unconstrained inflation and interest rates are

$$\widehat{\Pi^*_{H,t}} = \lambda_H^{SS} \rho^t \widehat{r_{H,0}} + \zeta_F^{SS} \rho^t \widehat{r_{F,0}} \text{ and } \widehat{i^*_{H,t+1}} = \gamma_\pi \widehat{\Pi^*_{H,t}},$$

where λ_H^{SS} and ζ_F^{SS} are Home inflation decision rule coefficients, for a regime with permanently slack Home and Foreign ZLB constraints (in such a regime the inflation decision rule has a zero intercept). A fundamentals-driven liquidity trap occurs when, for at least one of the two countries, the unconstrained nominal interest rate is negative at t=0, i.e. when (expressing the interest rate in deviation from steady state): $\widehat{i^*_{H,1}} < -(\Pi - \beta)/\Pi$ and/or $\widehat{i^*_{F,1}} < -(\Pi - \beta)/\Pi$. If only one of the countries has an unconstrained negative nominal interest rate at date t=0, then define T^* as the smallest date t>0 at which that country's unconstrained interest rate takes a non-negative value. If both countries have a negative unconstrained nominal interest rate at t=0, then let T_k^* be the smallest date t>0 at which country k's unconstrained interest rate takes a non-negative value, and define $T^* \equiv \max(T_H^*, T_F^*)$, i.e. T^* is the larger of the dates at which the two countries' unconstrained nominal interest rate cross the zero threshold. A fundamentals-driven liquidity trap equilibrium has the property that the ZLB constraint does not bind in either country at dates $t \geq T^*$. Thus, $\widehat{\Pi_{k,t}} = \widehat{\Pi^*_{k,t}}$ and $\widehat{i_{k,t+1}} = \widehat{i^*_{k,t+1}}$ hold for $t \geq T^*$. Inflation in periods $t < T^*$ is computed by iterating the two countries' Euler-Phillips equations backward. The known inflations rates $\widehat{\Pi_{k,T^*}} = \widehat{\Pi^*_{k,T^*}}$ and $\widehat{\Pi_{k,T^*+1}} = \widehat{\Pi^*_{k,T^*+1}}$ for k=H,F are used to back out $\widehat{\Pi_{k,T^*-1}}$ from country k=H,F date T^*-1 Euler-Phillips equation (B.18). Successive backward iterations allow to determine country k=H,F inflation for $0 \leq t < T^*$.

Flex-prices economy

In a flex-prices economy, real marginal cost is constant, and thus $\widehat{mc_{k,t}} = 0$, for k=H,F, where real marginal cost is given by (B.4). This condition, plus market clearing conditions (B.1) and the risk sharing condition (B.2) allows to solve for real quantities in the flex-prices economy:

$$\widehat{Y_{k,t}} = \tfrac{1}{D}\{(\phi-1)(2H-1)\Xi+H\}\cdot\widehat{\theta_{k,t}} - \tfrac{1}{D}(\phi-1)\Xi\cdot\widehat{\theta_{l,t}} +$$

$$\tfrac{1}{D}\{(\phi-1)\tfrac{H-1}{H}\Xi+1\}\cdot\widehat{G_{k,t}} + \tfrac{1}{D}\tfrac{H-1}{H}\Xi(\phi-1)\cdot\widehat{G_{l,t}} -$$

$$\tfrac{1}{D}((\phi-1)\Xi+1-\xi)\cdot(\widehat{\Psi_{kt}}-\widehat{\Psi_{lt}}), \quad \text{for } k,l\in\{\text{H,F}\},\ l\neq \text{k}; \qquad \text{(B.20)}$$

$$\widehat{C_{k,t}} = \tfrac{1}{D}\{(\phi-1)(H-1)\Xi+\xi H\}\cdot\widehat{\theta_{k,t}} + \tfrac{1}{D}\{(\phi-1)(H-1)\Xi+(1-\xi)H\}\cdot\widehat{\theta_{l,t}} -$$

$$\tfrac{1}{D}(H-1)\{(\phi-1)\Xi\tfrac{H-1}{H}+\xi\}\cdot\widehat{G_{k,t}} - \tfrac{1}{D}(H-1)\{(\phi-1)\Xi\tfrac{H-1}{H}+1-\xi\}\cdot\widehat{G_{l,t}} +$$

$$\tfrac{1}{D}\cdot\{\phi(H-1)\Xi+1-\xi\}\cdot(\widehat{\Psi_{kt}}-\widehat{\Psi_{lt}}) \ \text{for } k,l\in\{\text{H,F}\},\ k\neq l; \qquad \text{(B.21)}$$

$$NX_{k,t} = \tfrac{1}{D}H\Xi(\phi-1)\cdot(\widehat{\theta_{k,t}}-\widehat{\theta_{l,t}}) - \tfrac{1}{D}(H-1)\Xi(\phi-1)\cdot(\widehat{G_{k,t}}-\widehat{G_{l,t}}) -$$

$$\tfrac{1}{D}H(1-\xi)\{(\phi-1)2\xi+1\}\cdot(\widehat{\Psi_{k,t}}-\widehat{\Psi_{l,t}}) \quad \text{for } k,l\in\{\text{H,F}\},\ l\neq k. \qquad \text{(B.22)}$$

$$\widehat{q_t} = \tfrac{1}{D}H(\widehat{\theta_{H,t}}-\widehat{\theta_{F,t}}) + \tfrac{1}{D}(H-1)(\widehat{G_{H,t}}-\widehat{G_{F,t}}) + \tfrac{1}{D}\{(2\xi-1)(H-1)+1\}(\widehat{\Psi_{H,t}}-\widehat{\Psi_{F,t}}). \qquad \text{(B.23)}$$

Note that these expression can be obtained from the sticky-prices model solution (B.8)-(B.11), by setting an infinite Phillips curve slope, $\kappa=\infty$. Then terms involving inflation vanish in (B.8)-(B.11). The remaining terms in (B.8)-(B.11) (involving the exogenous shocks) correspond to the flex-prices model solution (B.20)-(B.23).

In a sticky-prices world, a monetary policy that fully stabilizes PPI inflation rate, at the central bank's inflation target, so that $\widehat{\Pi_{k,t}}=0\ \ \forall t$, entails that sticky-prices output, consumption, net exports and terms of trade equal the flex-prices counterparts of these variables. If inflation responses to exogenous shocks are sufficiently muted in a sticky-prices world, the transmission of those shocks to real activity will therefore resemble shock transmission under flexible prices.

For $\phi>1$ (as assumed in Sect. 5.1), the flex-prices model predicts negative transmission of productivity shocks to foreign output, but positive international transmission of government purchases shocks. For $\phi>1$, a positive productivity shock raises net exports in the country that receives the shock, while an increase in government purchases reduces net exports.

For $\phi=1.5$ and the other model parameters used in the simulations, the numerical solution of the flex-prices model is:

$$\widehat{Y_{k,t}} = 1.05 \cdot \widehat{\theta_{k,t}} - 0.05 \cdot \widehat{\theta_{l,t}} + 0.47 \cdot \widehat{G_{k,t}} + 0.03 \cdot \widehat{G_{l,t}} - 0.11 \cdot (\widehat{\Psi_{k,t}} - \widehat{\Psi_{l,t}}),$$

$$\widehat{C_{k,t}} = 0.83 \cdot \widehat{\theta_{k,t}} + 0.17 \cdot \widehat{\theta_{l,t}} - 0.42 \cdot \widehat{G_{k,t}} - 0.08 \cdot \widehat{G_{l,t}} + 0.21 \cdot (\widehat{\Psi_{k,t}} - \widehat{\Psi_{l,t}}),$$

$$NX_{k,t} = 0.10 \cdot (\widehat{\theta_{k,t}} - \widehat{\theta_{l,t}}) - 0.05 \cdot (\widehat{G_{k,t}} - \widehat{G_{l,t}}) + 0.22 \cdot (\widehat{\Psi_{k,t}} - \widehat{\Psi_{l,t}}),$$

$$\widehat{q_t} = -0.90 \cdot (\widehat{\theta_{H,t}} - \widehat{\theta_{F,t}}) + 0.45 \cdot (\widehat{G_{H,t}} - \widehat{G_{F,t}}) + 0.78 \cdot (\widehat{\Psi_{H,t}} - \widehat{\Psi_{F,t}}),$$

for $k,l \in \{\mathrm{H,F}\}$, $l \neq \mathrm{k}$.

Table 1. Baseline model with expectations-driven ZLB regimes: dynamic responses to persistent exogenous shocks

Horizon	i_H	Π_H	Y_H	C_H	i_F	Π_F	Y_F	C_F	q	S	NX_H
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(a) Binding Home and Foreign ZLB constraints											
Home productivity increase (1%)											
0	0.00	0.27	1.06	0.92	0.00	0.00	0.00	0.14	-1.06	-1.13	0.00
12	0.00	0.16	0.57	0.50	0.00	0.00	0.00	0.07	-0.57	-1.24	0.00
Home government purchases increase (1%)											
0	0.00	-0.14	0.47	-0.46	0.00	0.00	0.00	-0.07	0.53	0.56	0.00
12	0.00	-0.07	0.25	-0.25	0.00	0.00	0.00	-0.04	0.29	0.62	0.00
Home preference shock (1%)											
0	0.00	-0.26	-0.12	0.13	0.00	-0.02	0.06	-0.19	0.92	0.98	-0.13
12	0.00	-0.14	-0.07	0.07	0.00	-0.01	0.03	-0.10	0.50	1.08	-0.07
(b) Slack Home and Foreign ZLB constraints											
Home productivity increase (1%)											
0	-0.39	-0.26	0.94	0.82	0.00	0.00	0.00	0.12	-0.94	-0.89	0.00
12	-0.21	-0.14	0.51	0.44	0.00	0.00	0.00	0.07	-0.51	0.12	0.00
Home government purchases increase (1%)											
0	0.19	0.13	0.53	-0.41	0.00	0.00	0.00	-0.06	0.47	0.44	0.00
12	0.11	0.07	0.29	-0.22	0.00	0.00	0.00	-0.03	0.26	-0.06	0.00
Home preference shock (1%)											
0	0.36	0.24	-0.01	0.23	0.02	0.02	0.07	-0.17	0.82	0.76	-0.13
12	0.20	0.13	-0.01	0.12	0.01	0.01	0.04	-0.09	0.44	-0.10	-0.07

Notes: A model variant with expectations-driven ZLB regimes is considered. Trade elasticity: ϕ=1. Probability of remaining in the same ZLB regime next period: $p_{BB}=p_{SS}=0.95$. Autocorrelation of productivity, government purchases and preference shifter (Ψ): 0.95.

Panel (a): simultaneous Home and Foreign liquidity traps; Panel (b): slack Home and Foreign ZLB constraints.

Shock responses 0 and 12 periods (see Column labelled 'Horizon') after 1% innovations to Home productivity (θ_H), Home government purchases (G_H) and to the Home preference shifter (Ψ_H) are shown. The responses pertain to simulation runs without ZLB regime changes.

Endogenous variables: Home (H) and Foreign (F) nominal interest rates (i_H,i_F), producer price inflation (Π_H, Π_F), output (Y_H,Y_F), consumption (C_H,C_F), Home terms of trade (q), nominal exchange rate (S) and Home net exports/GDP ratio (NX_H). (A rise in 'q' is a Home terms of trade improvement and corresponds to an appreciation of the Home real exchange rate; a rise in 'S' is an appreciation of the Home nominal exchange rate.)

Responses of output, consumption, terms of trade and nominal exchange rate are reported as % deviations from the symmetric steady state. Responses of interest rates and inflation are reported as percentage point (ppt) per annum differences from steady state; responses of net exports/GDP are reported in ppt.

Table 2. Baseline model with Home and Foreign <u>fundamentals-driven</u> liquidity traps: baseline liquidity trap scenario and dynamic responses to persistent exogenous shocks

Horizon	i_H	Π_H	Y_H	C_H	i_F	Π_F	Y_F	C_F	q	S	NX_H
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(a) Baseline liquidity trap scenario (triggered by -9.89% Home and Foreign preference shock)											
0	0.00	-26.55	-13.60	-13.60	0.00	-26.55	-13.60	-13.60	0.00	0.00	0.00
5	0.00	-2.98	-1.70	-1.70	0.00	-2.98	-1.70	-1.70	0.00	0.00	0.00
12	0.15	0.10	-0.15	-0.15	0.15	0.10	-0.15	-0.15	0.00	0.00	0.00
						. . . .					
(b) Dynamic responses to shocks (shown as difference relative to baseline liquidity trap scenario)											
Home productivity increase (1%)											
0	0.00	-31.84	-14.90	-12.97	0.00	0.00	0.00	-1.94	14.90	22.82	0.00
5	0.00	-4.05	-1.30	-1.16	0.00	0.00	0.00	-0.17	1.30	22.82	0.00
12	-0.15	-0.21	0.49	0.43	0.00	0.00	0.00	0.06	-0.49	22.82	0.00
Home government purchases increase (1%)											
0	0.00	9.20	5.11	3.58	0.00	0.00	0.00	0.53	-4.11	-6.40	0.00
5	0.00	1.14	0.98	0.18	0.00	0.00	0.00	0.03	-0.21	-6.40	0.00
12	0.14	0.10	0.28	-0.23	0.00	0.00	0.00	-0.03	0.26	-6.44	0.00
Home preference shock (1%)											
0	0.00	14.80	7.37	6.73	0.00	1.20	0.67	1.31	-5.96	-9.35	-0.13
5	0.00	1.83	0.91	0.98	0.00	0.15	0.13	0.05	-0.21	-9.35	-0.10
12	0.27	0.18	-0.02	0.11	0.02	0.01	0.04	-0.09	0.46	-9.45	-0.07

Notes: A model variant with simultaneous Home and Foreign fundamentals-driven liquidity traps (12 periods) is considered. Trade elasticity: ϕ=1. Autocorrelation of productivity, government purchases and preference shifter (Ψ): 0.95.

Panel (a) reports the baseline liquidity trap scenario in which identical negative Home & Foreign preference shocks (-9.89%) induce Home and Foreign liquidity traps. Baseline paths (Panel (a)) of interest rates and inflation rates are shown in levels (*not* as deviations from steady state values) and expressed in percentage points (ppt) per annum; the baseline path of Home net exports/GDP ratio (NX_H) too is reported in ppt levels. Baseline paths of other variables (Panel (a)) represent % deviations from steady state.

Panel (b) reports dynamic responses after 0, 5 and 12 periods (see Column labelled 'Horizon') triggered by 1% innovations to exogenous variables. The exogenous innovations are added to the baseline liquidity trap scenario. Dynamic shock responses in Panel (b) are measured in the same units as the baseline paths (Panel (a)) and expressed as *differences* from the baseline paths shown in Panel (a). (Thus, interest rates and inflation rates responses in Panel (b) are expressed in ppt per annum and net exports are expressed in ppt.)

See Table 1 for definition of variables and other information.

Table 3. Model with expectations-driven ZLB regimes, *higher trade elasticity* (ϕ=1.5): impact responses to persistent exogenous shocks

Horizon i_H	Π_H	Y_H	C_H	i_F	Π_F	Y_F	C_F	q	S	NX_H
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(a) Binding Home and Foreign ZLB constraints										
Home productivity increase (1%)										
0.00	0.26	1.11	0.88	0.00	0.02	-0.05	0.18	-0.95	-1.00	0.11
Home government purchases increase (1%)										
0.00	-0.13	0.44	-0.45	0.00	-0.01	0.03	-0.09	0.47	0.50	-0.05
(b) Binding Home ZLB constraint, slack Foreign ZLB constraint										
Home productivity increase (1%)										
0.00	0.26	1.11	0.88	-0.02	-0.02	-0.06	0.17	-0.95	-1.02	0.11
Foreign productivity increase (1%)										
0.00	0.01	-0.05	0.16	-0.37	-0.25	0.99	0.79	0.85	0.78	-0.10
Home government purchases increase (1%)										
0.00	-0.13	0.45	-0.44	0.01	0.01	0.03	-0.09	0.48	0.51	-0.05
Foreign government purchases increase (1%)										
0.00	-0.01	0.02	-0.08	0.19	0.12	0.50	-0.39	-0.42	-0.39	0.05
(c) Slack Home and Foreign ZLB constraints										
Home productivity increase (1%)										
-0.37	-0.25	0.99	0.79	-0.02	-0.01	-0.05	0.16	-0.85	-0.79	0.10
Home government purchases increase (1%)										
0.19	0.12	0.50	-0.39	0.01	0.01	0.03	-0.08	0.43	0.40	-0.05

Notes: A model variant with expectations-driven ZLB regimes is considered. Same set-up as in Table 1, except that a higher trade elasticity is assumed: ϕ=1.5.
(Probability of remaining in the same ZLB regime next period: $p_{BB}=p_{SS}=0.95$. Autocorrelation of productivity, government purchases and preference shifter (Ψ): 0.95.)

Panel (a): simultaneous Home and Foreign liquidity traps; Panel (b): Home liquidity trap, but slack Foreign ZLB; Panel (c): slack Home and Foreign ZLB constraints.

Responses to 1% innovations to exogenous variables are reported.

See Table 1 for definitions of variables and other information.

Table 4. Model with <u>fundamentals-driven</u> liquidity traps, <u>*higher trade elasticity*</u> (ϕ=1.5): dynamic responses to persistent exogenous shocks.

Horizon	i_H	Π_H	Y_H	C_H	i_F	Π_F	Y_F	C_F	q	S	NX_H
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(a) Home and Foreign fundamental liquidity traps											
Home productivity increase (1%)											
0	0.00	-34.59	-17.63	-13.54	0.00	3.26	2.98	-1.11	16.81	26.22	-1.90
5	0.00	-4.16	-1.48	-1.14	0.00	0.17	0.21	-0.12	1.38	26.22	-0.16
12	-0.15	-0.20	0.52	0.41	-0.01	-0.01	-0.02	0.08	-0.44	26.22	0.05
Home government purchases increase (1%)											
0	0.00	9.93	5.85	3.75	0.00	-0.71	-0.72	0.38	-4.54	-7.20	0.51
5	0.00	1.17	1.02	0.19	0.00	-0.02	-0.04	0.02	-0.23	-7.20	0.03
12	0.14	0.09	0.26	-0.22	0.01	0.01	0.01	-0.04	0.24	-7.22	-0.03
(b) Home fundamental liquidity trap (Foreign ZLB constraint does not bind)											
Home productivity increase (1%)											
0	0.00	-34.40	-17.41	-13.70	1.20	0.80	1.27	-2.43	15.23	23.99	-1.72
5	0.00	-4.15	-1.47	-1.15	0.13	0.09	0.13	-0.19	1.30	24.74	-0.15
12	-0.15	-0.20	0.52	0.41	-0.01	-0.01	-0.02	0.08	-0.44	24.81	0.05
Foreign productivity increase (1%)											
0	0.00	-1.49	-0.85	-0.47	-0.46	-0.30	1.08	0.70	1.58	1.87	-0.18
5	0.00	-0.18	-0.14	0.05	-0.39	-0.26	0.80	0.61	0.76	1.34	-0.09
12	-0.01	-0.01	-0.03	0.09	-0.27	-0.18	0.55	0.44	0.47	0.75	-0.05
Home government purchases increase (1%)											
0	0.00	9.91	5.81	3.78	-0.34	-0.23	-0.36	0.67	-4.21	-6.73	0.48
5	0.00	1.17	1.02	0.19	-0.03	-0.02	-0.03	0.03	-0.22	-6.95	0.02
12	0.14	0.09	0.26	-0.22	0.01	0.01	0.01	-0.04	0.24	-6.99	-0.03
Foreign government purchases increase (1%)											
0	0.00	0.52	0.30	0.14	0.23	0.16	0.47	-0.37	-0.68	-0.77	0.08
5	0.00	0.06	0.05	-0.04	0.19	0.13	0.38	-0.31	-0.37	-0.50	0.04
12	0.01	0.01	0.01	-0.04	0.14	0.09	0.26	-0.22	-0.24	-0.21	0.03

Notes: A model variant with fundamentals-driven liquidity traps (12 periods) is considered. Same set-up as in Table 2, except that a higher trade elasticity is assumed: ϕ=1.5. (Autocorrelation of productivity, government purchases and the preference shifter (Ψ): 0.95.)

Panel (a) assumes simultaneous Home and Foreign fundamentals-driven liquidity traps (caused by -9.89% Home and Foreign preference shock).

Panel (b) assumes a fundamentals-driven liquidity trap just in the Home country (caused by -11.09% Home preference shock; there is no Foreign preference shock).

The Table shows dynamic responses of 1% innovations to exogenous variables that are added to baseline liquidity trap scenarios (the baseline scenario for Panel (a) is identical to the baseline scenario in Table 2; baseline scenario for Panel (b) is not reported); dynamic shock responses are expressed as differences from the respective baseline liquidity trap scenarios.

See Table 2 for definition of variables and other information.

Table 5. Model with expectations-driven ZLB regimes, *higher trade elasticity* (ϕ=1.5): impact responses to *less persistent exogenous shocks* (autocorrelation: 0.5)

	i_H	Π_H	Y_H	C_H	i_F	Π_F	Y_F	C_F	q	S	NX_H
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(a) Binding Home and Foreign ZLB constraints											
Home productivity increase (1%)											
	0.00	-1.89	-0.46	-0.36	0.00	0.04	0.05	-0.05	0.42	0.90	-0.05
Home government purchases increase (1%)											
	0.00	0.94	1.23	0.18	0.00	-0.02	-0.03	0.02	-0.21	-0.45	0.02
(b) Binding Home ZLB constraint, slack Foreign ZLB constraint											
Home productivity increase (1%)											
	0.00	-1.89	-0.46	-0.36	0.03	0.02	0.04	-0.06	0.41	0.88	-0.05
Foreign productivity increase (1%)											
	0.00	-0.03	-0.05	0.04	-1.30	-0.86	0.42	0.33	0.38	0.18	-0.04
Home government purchases increase (1%)											
	0.00	0.94	1.23	0.18	-0.01	-0.01	-0.02	0.03	-0.20	-0.44	0.02
Foreign government purchases increase (1%)											
	0.00	0.02	0.02	-0.02	0.65	0.43	0.79	-0.16	-0.19	-0.09	0.02
(c) Slack Home and Foreign ZLB constraints											
Home productivity increase (1%)											
	-1.30	-0.86	0.42	0.33	-0.03	-0.02	-0.03	0.06	-0.37	-0.16	0.04
Home government purchases increase (1%)											
	0.65	0.43	0.79	-0.16	0.01	0.01	0.02	-0.03	0.18	0.08	-0.02

Notes: A model variant with expectations-driven ZLB regimes is considered. Same set-up as in Table 1, except that a higher trade elasticity (ϕ=1.5) is assumed, and that productivity and government purchases are less persistent (autocorrelation: 0.5). (Probability of remaining in the same ZLB regime next period: p_{BB}=p_{SS}=0.95.)

Panel (a): simultaneous Home and Foreign liquidity traps; Panel (b): Home liquidity trap, but slack Foreign ZLB; Panel (c): slack Home and Foreign ZLB constraints.

Responses to 1% innovations to exogenous variables are reported.

See Table 1 for definitions of variables and other information.

Table 6. Model with <u>fundamentals-driven</u> liquidity traps, *<u>higher trade elasticity</u>* (ϕ=1.5): impact responses to *<u>less persistent exogenous shocks</u>* (autocorrelation: 0.5)

i_H	Π_H	Y_H	C_H	i_F	Π_F	Y_F	C_F	q	S	NX_H
(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
(a) Home and Foreign fundamental liquidity traps										
Home productivity increase (1%)										
0.00	-2.03	-0.55	-0.43	0.03	0.05	0.07	-0.05	0.50	1.02	-0.06
Home government purchases increase (1%)										
0.00	1.01	1.27	0.21	-0.02	-0.02	-0.03	0.03	-0.25	-0.50	0.03
(b) Home fundamental liquidity trap (Foreign ZLB constraint does not bind)										
Home productivity increase (1%)										
0.00	-2.03	-0.55	-0.43	0.03	0.02	0.04	-0.07	0.48	0.99	-0.05
Foreign productivity increase (1%)										
0.00	-0.04	-0.06	0.04	-1.18	-0.79	0.44	0.34	0.40	0.22	-0.05
Home government purchases increase (1%)										
0.00	1.01	1.27	0.21	-0.02	-0.01	-0.02	0.04	-0.24	-0.49	0.03
Foreign government purchases increase (1%)										
0.00	0.02	0.03	-0.02	0.59	0.39	0.78	-0.17	-0.20	-0.11	0.02

Notes: A model variant with fundamentals-driven liquidity traps (12 periods) is considered. Same set-up as in Table 2, except that a higher trade elasticity (ϕ=1.5) is assumed, and that productivity and government purchases are less persistent (autocorrelation: 0.5). The liquidity traps are generated by persistent one-time preference shocks (autocorrelations of the preference shocks: 0.95).

Panel (a) assumes a simultaneous Home and Foreign fundamentals-driven liquidity trap (caused by -9.89% Home and Foreign preference shock).

Panel (b) assumes a fundamentals-driven liquidity trap (12 periods) just in Home country (caused by -11.09% Home preference shock).

The Table shows dynamic responses of 1% innovations to exogenous variables that are added to baseline liquidity trap scenarios (the baseline scenario for Panel (a) is identical to the baseline scenario in Table 2; baseline scenario for Panel (b) is not reported); dynamic shock responses are expressed as differences from the respective baseline liquidity trap scenarios.

See Table 2 for definition of variables and other information.