CHAPTER III

WORLD BUSINESS CYCLES AND INCOMPLETE INTERNATIONAL ASSET MARKETS

1. Introduction

Standard neoclassical models of international economic fluctuations predict that private consuption is strongly correlated across countries (Scheinkman (1984), Backus, Kehoe & Kydland (1989), Crucini (1989), Baxter & Crucini (1989)). This prediction is incompatible with the low cross country-correlations of consumption which are observed empirically. While existing international theories assume complete international asset markets, this paper develops a model with incomplete international asset markets and it shows that in such a model the cross-country correlation of consumption can be markedly smaller than in models with complete asset markets.

The main reason why previous international models have assumed complete asset markets is presumably tractability: with complete asset markets, competitive equilibria are Pareto optimal and hence they can be determined by solving a social planning problem. Simple numerical methods for solving such a problem are available (c.f. Rebelo (1989 a,b); King, Plosser & Rebelo (1988); Backus, Kehoe & Kydland (1989)). With incomplete asset markets, the equivalence between competitive equilibra and Pareto optima breaks down. I present a method for solving a stochastic dynamic general equilibrium model with incomplete asset markets which is (almost) as simple as the methods used by previous researchers to solve models with complete asset markets.

The paper also argues that a model with *additive* technology shocks which do not affect marginal factor productivities is better able to explain the observed positive correlations of investment and output across countries than standard models in which *multiplicative* shocks to total (and marginal) factor productivities are the source of fluctuations.[1] The equalization of the expected marginal products of capital in different countries which takes place in an integrated world capital market implies that when labor supply elasticities are low, then in the absence of shocks to the marginal product of capital schedule, capital and output are positively correlated across countries. A positive additive shock to the technology of one of the countries induces increases in the capital stocks of all countries in the world (provided the shock leads to a fall in the world interest rate). In contrast, a persistent positive multiplicative shock to the technology of one of the countries increases the marginal product of capital in that country; hence it induces an increase in investment and output in that country and - unless there are strong technological 'spill-overs' between the two countries - it leads to a fall in the other country's investment and output; unless multiplicative technology shocks are strongly correlated across countries, investment tends to be negatively correlated across countries.

In comparison, even additive technology shocks which are not correlated

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[1]Specifically, I consider production functions of the type $y=\theta*f(K)-g$, where θ and g are random variables representing *multiplicative* and *additive* technology shocks respectively, while K denotes physical capital.

across countries lead to positive cross-country correlations of investment (and output), provided that labor supply elasticities are low.

One possible interpretation of the additive technology shocks is as government consumption which is financed through lump-sum taxes. My solution method for the model with incomplete asset markets consists in approximating the agents' optimal decision rules by linear functions. Using these linear decision rules and imposing market clearing allows one to solve for an 'approximate' equilibrium.

Section 2 presents the two country model with incomplete asset markets and discusses the method used to numerically solve this model. Section 3 compares the behavior of the model to the behavior which would obtain in the presence of complete asset markets. Stylized facts about international business cycles are described in section 4. I use simulations to explore the properties of the model; simulation results are presented in section 5. Section 6 summarizes the findings obtained in this chapter.

2. The Model

2.1 *Preferences and Technologies*

I consider a world consisting of two countries (i=1,2); each country is inhabited by one consumer (the model can easily be extended to allow for different population sizes in the two countries). There exists a unique good in this world, which is produced and consumed by both countries. This good can also be used as an investment good.

Country i's intertemporal preferences are represented by

$$V_t^i=u^i(c_t^i,L_t^i,\tau_t^i)+\beta(c_t^i,L_t^i,\tau_t^i)*E_t\{V_{t+1}^i\}; \tag{1}$$

here V_t^i is i's expected lifetime utility in period t; c_t^i is i's consumption at date t, L_t^i denotes the number of hours worked by country i and τ_t^i is a random variable which represents an exogenous taste shock. E_t denotes expectations conditional on all informations available in period t; $0<\beta(c_t^i,L_t^i,\tau_t^i)\equiv 1/(1+b(c_t^i,L_t^i,\tau_t^i))<1$, where $b(c_t^i,L_t^i,\tau_t^i)$ is the agent's subjective rate of time preference between periods t and t+1;

I assume that the subjective rate of time preference between periods t and t+1 is a function of c_t^i, L_t^i and τ_t^i.[2] This assumption allows to guarantee the existence of a steady state which is unique (at least locally) as well as the existence of an equilibrium which involves stationary fluctuations around that steady state. This makes it possible to apply standard techniques used in business cycle analysis to solve for this equilibrium and to explore its properties using simulations.

Note that in the special case (which is usually considered in closed economy macro-economics) where β is constant, (1) gives the 'standard' intertemporal utility function $V_t=E_t\sum_{\tau=0}^{\tau=\infty}\beta^{\tau}*u_{t+\tau}$.

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[2] The idea that the rate of time preference depends on consumption has a long history (for an early discussion, see Fisher (1930); Obstfeld (1989) gives a recent discussion). Examples of its use in the international economics literature are Calvo & Findlay (1978), Obstfeld (1981 a,b), Mendoza (1989 a,b). Other examples can be found in Beals & Koopmans (1969), Lucas & Stokey (1984), Epstein (1987).

The period utility function u is increasing in consumption and decreasing in labor and it is strictly concave in both arguments; the function β is assumed to have properties which guarantee that V_t is increasing and concave in current and future consumption.

Country i has a technology which is described by:

$$y_t^i=\theta_t^i f^i(K_t^i,L_t^i)-g_t^i; \tag{2}$$

$f^i()$ is a function with the usual neoclassical properties. K_t^i is country i's capital stock in period t, while θ_t^i and g_t^i are stochastic shocks which are assumed to be stationary (the model abstracts from growth). In standard business cycle theories, *multiplicative* technology shocks (θ_t^i) are the only source of economic fluctuations. Note that while θ_t^i affects the marginal product of capital ($\theta_t^i * f^{i\prime}(K_t^i)$), the *additive* shock g_t^i does not. This is why - as simulations of the model show - the multiplicative shock has a much stronger impact on investment than the additive shock. g_t^i can either be interpreted as a technology shock or as government consumption which is financed by a lump-sum tax. In that case, the output of country 'i' is given by

$$q_t^i \equiv \theta_t^i f^i(K_t^i,L_t^i) \tag{3}$$

and y_t^i (as defined in (2)) is the output of country i which is available to the private sector. Note that the L_t^i variable which appears in the production function and in the utility function (1) is the same: all labor supplied by country i has to be used for production in that same country (i.e. labor cannot move internationally).

The law of motion of the capital stock in country 'i' is:

$$K^i_{t+1}=(1-d)*K^i_t+I^i_t \ ; \quad (4)$$

here d is a fixed rate of depreciation, while I^i_t is gross investment made in period t. Hence country i's capital stock in period t+1 is 'predetermined' in t: country i decides in t what its capital stock in the following period will be.

2.2 **Unconditional Borrowing and Lending**

In period t, country i can make (or receive) one period loans at the real rate r_t: if 'i' makes a loan of A^i_t units of the good in period t, it gets back $(1+r_t)*A^i_t$ units in period t+1 (the model can be extended to allow for loans of different maturities). $A^i_t>0$ means that the country is a lender and $A^i_t<0$ means that it is a borrower in period t.

Country i's period t budget constraint is:

$$c^i_t+K^i_{t+1}+A^i_t=\theta^i_t f^i(K^i_t,L^i_t)-g^i_t+(1-d)*K^i_t+(1+r_{t-1})*A^i_{t-1} \ . \quad (5\ a)$$

In order to rule out Ponzi schemes, the following constraint is imposed:[3]

$$-Z \le A^i_t \quad \text{for all } t \ . \quad (5\ b)$$

Z is a large positive number.

The real rate r_t is set in period t and hence it can depend on informations available in period t, but not on informations which become available in

[3]See Sargent (1987), p.364, for an example of the use of this constraint.

t+1. Hence there are no insurance markets in the present model.[4]

The decision problem of a resident of country i is to maximize the lifetime utility defined in (1) subject to the constraints that (5 a) and (5 b) hold in all periods.

The equilibria on which I focus involve small fluctuations of all variables of the model around a steady state. In these equilibria, the constraint (5 b) never binds and therefore the solution to the decision problem of a resident of country i satisfies the following familiar Euler equations:[5]

$$u^i_{1,t}+\beta^i_{1,t}*E_tV^i_{t+1}=(1+r_t)*\beta^i_t*E_t\{u^i_{1,t+1}+\beta^i_{1,t+1}*E_{t+1}V^i_{t+2}\} \quad \text{for } i=1,2 . \tag{6}$$

$$u^i_{1,t}+\beta^i_{1,t}E_tV^i_{t+1}=\beta^i_tE_t\left\{[\theta^i_{t+1}f^i_{1,t+1}+(1-d)]*[u^i_{1,t+1}+\beta^i_{1,t+1}E_{t+1}V^i_{t+2}]\right\} \quad \text{for } i=1,2. \tag{7}$$

$u^i_{s,t}$ and $f^i_{s,t}$ denote the derivatives of $u^i(c^i_t,L^i_t,\tau^i_t)$ and $f^i(K^i_t,L^i_t)$ with

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[4] Risk-averse agents whose incomes fluctuate in an unpredictable way due to exogenous factors, have an incentive to issue or purchase assets which provide them with insurance against these fluctuations. The absence of such insurance can be justified by assuming that the income of a given country (or the exogenous factors which lead to fluctuations in that income) cannot be observed by agents in the other country (or only at a prohibitive cost).

[5] To motivate (6), note that country i behaves optimally, its expected lifetime utility does not change if it reduces its consumption in period t by an infinitesimal amount ε in order to buy a bond which has a return r_t in period t+1 and if it uses the proceeds from this investment to increase his consumption at date t+1 by $(1+r_t)*\varepsilon$. A similar reasoning shows that when the agent behaves optimally, his expected lifetime utility does not change if he reduces his consumption in period t by an infinitesimal amount ε in order to increase his capital stock in t+1 by ε and if he consumes the proceeds from this additional investment (so that the capital stock in t+2 and in subsequent periods is unchanged). This explains (7).

respect to the s^{th} argument of these functions.

In addition we have that in every period (and for all realizations of the exogenous shocks) country i equates its marginal rate of substitution between labor and consumption to the marginal product of labor and therefore:

$$\{u^i_{2,t}+\beta^i_{2,t}*E_tV^i_{t+1}\}+[\theta^i_t*f^i_{2,t}]*\{u^i_{1,t}+\beta^i_{1,t}*E_tV^i_{t+1}\}=0. \quad (8)$$

Given exogenous processes $\{\theta^1_t,\theta^2_t,g^1_t,g^2_t,\tau^1_t,\tau^2_t\}$,[6] an *equilibrium with incomplete asset markets* is a set of stochastic processes for the endogenous variables of the model $\{c^1_t,c^2_t,L^1_t,L^2_t,V^1_t,V^2_t,K^1_t,K^2_t,A^1_t,A^2_t,r_t\}$ which satisfy (1), (5 a), (6), (7) and (8) as well as the condition that the loan market clears:[7]

$$A^1_t+A^2_t=0 \qquad \text{for all } t . \quad (9)$$

A *steady state with incomplete asset markets* is an *equilibrium with incomplete asset markets* in which all exogenous and endogenous variables are constant. Given constant values $(\theta^1,\theta^2,g^1,g^2,\tau^1,\tau^2)$ for the exogenous shocks, a steady state is thus a solution of (1), (5 a), (6), (7), (8) and (9) in which the endogenous variables of the model are constant.

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[6]And assuming that (5 b) never binds.

[7]By Walras' law, market clearing in the loan market implies that the goods market clears as well.

Given constant values $(\theta^1,\theta^2,g^1,g^2,\tau^1,\tau^2)$ for the exogenous shocks, a steady state with incomplete asset markets can thus be described by a vector $(c^1,c^2,L^1,L^2,V^1,V^2,K^1,K^2,A^1,A^2,r)$ which satisfies the following equations:

$$V^i = u^i/(1-\beta^i) \quad \text{for } i=1,2; \tag{10 a}$$

$$c^i=\theta^i*f^i-d*K^i-g^i+r*A^i \quad \text{for } i=1,2; \tag{10 b}$$

$$(1+r)*\beta^i=1 \quad \text{for } i=1,2; \tag{10 c}$$

$$(u_2^i+\beta_2^i*V^i)+[\theta^i*f_2^i]*(u_1^i+\beta_1^i*V^i)=0 \quad \text{for } i=1,2; \tag{10 d}$$

$$r+d=\theta^i*f_1^i \quad \text{for } i=1,2; \tag{10 e}$$

$$A^1+A^2=0 \ . \tag{10 f}$$

Hence a steady state is the solution to a system of 11 equations in 11 unknowns. Note that if β is constant there does not exist a unique steady state because then the two equations in (10 c) reduce to $(1+r)*\beta=1$.

While working on this paper, I learned about recent work by Conze & Scheinkman (1990) and Stockman & Tesar (1990) which gets results which (with respect to explaining low cross-country correlations of consumption) are similar to those which I present. Conze & Scheinkman (1990) too consider a model with incomplete international asset markets. In their paper,[8] the only way in which agents can protect themselves against fluctuations in their labor productivity is by holding a durable asset ('money'). Agents must hold a positive amount of this asset, i.e. there is no borrowing at all in Conze & Scheinkman (1990).
An important difference between their work and my paper is that my solution method makes it possible to consider a model which is less stylized than their's; for example, my model allows for physical capital,

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[8] As in Scheinkman & Weiss (1986).

several types of exogenous shocks etc.

Stockman & Tesar (1990) present a model with complete international asset markets in which non-traded consumption goods and country-specific taste shocks break the close cross-country correlation of consumption.[9]

As a first pass, and in order to focus on the effects of limitations on asset markets, the present paper simply abstracts from non-traded consumption goods.

2.3. *The Solution Method*

An approximate solution to the model can be obtained by deriving a linear approximation to the equations listed in the definition of the equilibrium and by solving the resulting system of expectational difference equations. For the case where the exogenous shocks have small variances, and where there would exist a locally unique steady state for constant values of the exogenous variables equal to the unconditional means of the random shocks θ_t^1, θ_t^2, g_t^1, g_t^2, τ_t^1 and τ_t^2 Woodford (1986) [10] provides a rigorous justification for conducting the linear approximation around this steady state.

Hence I assume that when the exogenous shocks are equal to their unconditional means, a (locally) unique steady state exists ; I expand the equations listed in the above definition of an equilibrium in Taylor series around this steady state and I keep the linear terms of these expansions.

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[9]Their empirical analysis shows that allowing for non-traded goods in a model with complete asset markets is not sufficient to reduce the cross-country correlation of consumption to the correlations which are observed empirically; this is why Stockman & Tesar resort to country specific taste shocks.

[10]See also the discussion in Rotemberg & Woodford (1989).

In what follows $\Delta\hat{x}_t$ denotes the percentage deviation of a variable x_t from its steady state.[11]

Given exogenous forcing processes for $\{\Delta\hat{\theta}^1_t, \Delta\hat{\theta}^2_t, \Delta\hat{g}^1_t, \Delta\hat{g}^2_t, \Delta\hat{\tau}^1_t, \Delta\hat{\tau}^2_t\}$, *an approximate equilibrium with incomplete asset markets* is a set of stochastic processes for the endogenous variables $\{\Delta\hat{c}^1_t, \Delta\hat{c}^2_t, \Delta\hat{L}^1_t, \Delta\hat{L}^2_t, \Delta\hat{V}^1_t, \Delta\hat{V}^2_t, \Delta\hat{K}^1_t, \Delta\hat{K}^2_t, \Delta\hat{A}^1_t, \Delta\hat{A}^2_t, \Delta\hat{r}_t\}$ which satisfy the linearized versions of the equations listed in the definition of an equilibrium.

In what follows, I focus on 'approximate equilibria'.

Keeping the linear terms of the Taylor expansion of the equations listed in the definition of an equilibrium, we obtain (after a few substitutions) a system of equations which can be written as:

$$E_t h_{t+1} = G_0 * h_t + G_1 * z_t + G_2 * E_t z_{t+1} \, , \tag{11}$$

where $h_t \equiv (\Delta\hat{r}_{t-1}, \Delta\hat{K}^1_t, \Delta\hat{K}^2_t, \Delta\hat{A}^1_t, \Delta\hat{V}^1_t, \Delta\hat{L}^1_t, \Delta\hat{c}^1_t, \Delta\hat{V}^2_t, \Delta\hat{L}^2_t, \Delta\hat{c}_t)'$ and $z_t \equiv (\Delta\hat{\theta}^1_t, \Delta\hat{\theta}^2_t, \Delta\hat{g}^1_t, \Delta\hat{g}^2_t, \Delta\hat{\tau}^1_t, \Delta\hat{\tau}^2_t)'$. G_0 is a 10x10 matrix, while G_1 and G_2 are 10x6 matrices.

The first four elements of the vector h_t are 'predetermined' at date t (i.e. they are known at t-1), while the remaining elements are 'non-predetermined'. As shown by Blanchard & Kahn (1980), a necessary and

[11] With the following exception(s): in order to allow for cases where steady state asset holdings are zero and where $g^i=0$, $\Delta\hat{A}^i_t$ and $\Delta\hat{g}^i_t$ are defined by dividing the differences between A^i_t and g^i_t and their respective steady states by y^i.

sufficient condition for a model like (11) to have a unique stationary solution is that the number of eigenvalues of the matrix G_0 outside the unit circle equals the number of non-predetermined variables.

For a simple version of the model where labor supplies are fixed, $\partial\beta/\partial c<0$ can be shown to be a necessary condition for guaranteeing the existence of a unique stationary solution.[12]

The approximation method used here is equivalent to a method which consists in substituting agent i's budget constraint (5a) into his lifetime utility function (1) and expanding the resulting function in a second order Taylor expansion in all its arguments (for all dates and states) around their respective steady state values; maximizing the quadratic function which this expansion yields, gives a set of first-order conditions which is equivalent to (6)-(8).

Christiano (1990) shows that for a one sector neoclassical growth model this linear-quadratic approximation method performs strikingly well.[13]

Linearizing (6) and (7), we get that

$$\Delta r_t=f_1^i*E_t\Delta\theta_{t+1}^i+\theta^i*f_{1,1}^i*\Delta K_{t+1}^i+\theta^i*f_{1,2}^i*E_t\Delta L_{t+1}^i \quad . \tag{12}$$

This equation says that each country equates the expected marginal product of its capital stock to the risk-free interest rate. When labor supplies are variable or when the model is subjected to *multiplicative* stochastic

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[12] In the simulations described below, I assume that $\beta(c,L,\tau)=\beta(u(c,L,\tau))$. Numerical 'experiments' with many different parameter values suggest that unless $\partial\beta/\partial u<0$ holds, the Blanchard & Kahn condition for the existence of a unique stationary solution is not likely to be satisfied.

[13] See Taylor & Uhlig (1990) for an overview over different methods for solving the neoclassical growth model.

technology shocks, the marginal product of capital is random; hence (12) shows that the risk premium associated with this randomness is neglected when the linearization method is used. Note however that by making the variance of the exogenous shocks sufficiently small, the risk premium can be made arbitrarily small.[14]

The solution of (11) can easily be determined using Blanchard & Kahn (1980). It has the property that - as ΔA^1_{t-1} is a predetermined variable at date t - the decisions which the two countries make in equilibrium depend on ΔA^1_{t-1} (i.e. on the distribution of financial assets between the two countries). That country i's date t consumption and labor supply choices depend on ΔA^i_{t-1} is not surprising, as these choices clearly depend on i's wealth. As the marginal product of capital depends on the labor input, this implies that physical investment decisions in the two countries are also affected by the distribution of financial wealth between the two countries.

An important special case in which (in the approximate equilibrium) the distribution of financial wealth does not affect the behavior of world-wide aggregates of consumption, labor supplies and the world capital stock

[14] To see this , let $m_{t,t+1}$ denote the marginal rate of substitution between consumption at dates t and t+1 ; from (6) and (7) we get that $E_t\theta^i_{t+1}*f'(K^i_{t+1})=(r_t+d)-cov(\theta^i_{t+1}*f'(K^i_{t+1}),m^i_{t,t+1})/E_t m^i_{t,t+1}$.

Using the Cauchy-Schwartz inequality, we get $|E_t\theta^i_{t+1}*f'(K^i_{t+1})-(r_t+d)|\le\{Var(\theta^i_{t+1}*f'(K^i_{t+1}))*Var(m^i_{t,t+1})\}^{0.5}/E_t m^i_{t,t+1}$.
By making the variances of the exogenous shocks sufficiently small, we can make the variances on the right-hand side of this inequality arbitrarily small.

occurs in the symmetric case when the two countries have identical preferences and technologies [15]: when this condition is satisfied, the coefficients of the linearized versions of the equilibrium conditions (1), (5a), (6), (7) and (8) are the same for i=1,2. Summing each of these linearized equations over i=1,2 gives a system of equations in the world capital stock, world consumption and world labor supply (the net asset positions of the two countries cancel out when the sum over i=1,2 is taken). This system uniquely determines the world-wide agregates of these variables.

3. Comparison of International Business Cycles With Complete and With Incomplete International Asset Markets

In the presence of complete asset markets, agents can at each date t trade in claims to units of consumption at date t+s, conditional on each possible 'history' of the realizations of the exogenous shocks between dates t and t+s. Let $x_t \equiv (\theta_t^1, \theta_t^2, g_t^1, g_t^2, \tau_t^1, \tau_t^2)$ and let $X^t \equiv \{\ldots, x_{t-2}, x_{t-1}, x_t\}$ denote the history of the shocks which have affected the two countries up until date t. In what follows, $q_t(X^s)$ with $s \geq t$ denotes the price at date t (in terms of output at t) of a claim to one unit of output to be delivered at date s if the history X^s occurs and $w_t^i(X^s)$ is the number of such claims which country i holds at the end of period t. Given X^t, the date t budget of country 'i' is thus:

[15] What is needed is identical u, β and f functions for the two countries and identical steady state values of the exogenous shocks affecting the two countries (and hence identical steady state values of the decision variables of the two countries).

$$c_t^i+K_{t+1}^i+\sum_{s=t+1}^{\infty}\int q_t(X^s)w_t^i(X^s)dX^s=\theta_t^i f^i(K_t^i,L_t^i)-g_t^i+w_{t-1}^i(X^t)+\sum_{s=t+1}^{\infty}\int q_t(X^s)w_{t-1}^i(X^s)dX^s. \qquad (13)$$

The decision problem of country 'i' is to maximize its expected life-time utility subject to the restriction that this budget constraint holds in all dates and for all possible histories.

In the presence of complete date and state contingent asset markets, we can characterize equilibria by using the equivalence between competitive equilibria and Pareto optima. Pareto optima can be found by maximizing a weighted sum of the expected lifetime utility levels of the residents of countries 1 and 2; therefore we can characterize equilibria by imagining that in some 'initial' time period t=0, the following problem is solved:

$$\text{Maximize} \qquad W_0\equiv\lambda * V_0^1+(1-\lambda) * V_0^2 \qquad (14)$$

[16]

subject to the constraint that the world resource constraint holds in all periods $t\geq0$:

$$c_t^1+c_t^2+K_{t+1}^1+K_{t+1}^2=\theta_t^1 f(K_t^1,L_t^1)+\theta_t^2 f(K_t^2,L_t^2)-g_t^1-g_t^2+(1-d) * K_t^1+(1-d) * K_t^2 \quad . \qquad (15)$$

Differentiating the objective function of the social planner with respect to the consumptions of the two countries gives the following condition:

$$B_{t-1} * \lambda * \{u_{1,t}^1+\beta_{1,t}^1 * E_t V_{t+1}^1\}=(1-\lambda) * \{u_{1,t}^2+\beta_{1,t}^2 E_t V_{t+1}^2\}, \qquad (16)$$

[16] λ and $(1-\lambda)$ are 'welfare' weight attached to the residents of the two countries. These weights reflect the wealth of the two countries (see Backus, Kehoe & Kydland (1989) and Rebelo (1988 a,b)).

where $B_{t-1} \equiv B_{t-2} * \beta^1(c^1_{t-1}, L^1_{t-1}, \tau^1_{t-1}) / \beta^2(c^2_{t-1}, L^2_{t-1}, \tau^2_{t-1})$. (17)

Hence we see that with complete asset markets, weighted marginal utilities of consumption of the two countries are equated in period t, where the weights attached to the marginal utilities reflect the 'welfare weights' (λ, $1-\lambda$) in the above social planning problem, as well as the consumptions of the countries *before* date t.

Note that in the standard case where β is constant (16) becomes: $\mu * u^1_{1,t} = u^2_{1,t}$, where μ is a constant. If the utility function u is separable in consumption and its other arguments, then $\mu * u^1_{1,t} = u^2_{1,t}$ implies that (locally) consumption is perfectly correlated across countries.

(16) and (17) imply that with complete markets, the marginal rates of substitution of the two countries between consumption at dates t and t+1 are equated, and that for all possible realizations of the exogenous shocks in the two periods. In contrast to this, with incomplete markets, marginal rates of substitution between t and t+1 are in general not equated across the two countries, although they are in expected value: (6) implies that $1/(1+r_t) = \beta^1_t * [E_t\{u^1_{1,t+1} + \beta^1_{1,t+1} * E_{t+1} V^1_{t+2}\}] / [u^1_{1,t} + \beta^1_{1,t} * E_t V^1_{t+1}] =$

$$\beta^2_t * [E_t\{u^2_{1,t+1} + \beta^2_{1,t+1} * E_{t+1} V^2_{t+2}\}] / [u^2_{1,t} + \beta^2_{1,t} * E_t V^2_{t+1}] .$$

Equation (7) and (8) are clearly valid first-order condition for the Pareto problem.

Given a welfare weight λ and exogenous shock processes $\{\theta^1_t, \theta^2_t, g^1_t, g^2_t, \tau^1_t, \tau^2_t\}$ an *equilibrium with complete asset markets* is a set of stochastic processes for the endogenous variables $\{c^1_t, c^2_t, L^1_t, L^2_t, V^1_t, V^2_t, K^1_t, K^2_t\}$ which satisfy (1), (7), (8), (15), (16) and (17).

One can easily show that the model with complete asset markets has the same steady state consumptions, labor supplies and capital stocks (and hence the same steady state interest rate) as the model with incomplete asset markets. Linearizing the equations listed in the definition of an equilibrium with complete asset markets around a steady state gives (after several substitutions) a system of equations of the form:

$$E_t e_{t+1} = H_0 * e_t + H_1 * z_t + H_2 * E_t z_{t+1} \ , \tag{18}$$

where $e_t \equiv (\Delta\hat{B}_{t-1}, \Delta\hat{K}^1_t, \Delta\hat{K}^2_t, \Delta\hat{V}^1_t, \Delta\hat{L}^1_t, \Delta\hat{c}^1_t, \Delta\hat{V}^2_t, \Delta\hat{L}^2_t, \Delta\hat{c}^2_t)'$ and $z_t \equiv (\Delta\hat{\theta}^1_t, \Delta\hat{\theta}^2_t, \Delta\hat{g}^1_t, \Delta\hat{g}^2_t, \Delta\hat{\tau}^1_t, \Delta\hat{\tau}^2_t)'$. H_0 is a 9x9 matrix, while G_1 and G_2 are 9x6 matrices.

Linearizing (7), (16) and (17) shows that in an approximate equilibrium with complete asset markets, the expected marginal products of capital are equated across countries (as is the case with incomplete markets, see (12)):

$$f^1_1 * E_t \Delta\theta^1_{t+1} + \theta^1 * f^1_{11} * \Delta K^1_{t+1} + \theta^1 * f^1_{12} * E_t \Delta L^1_{t+1} = f^2_1 * E_t \Delta\theta^2_{t+1} + \theta^2 * f^2_{11} * \Delta K^2_{t+1} + \theta^2 * f^2_{12} * E_t \Delta L^2_{t+1}. \tag{19}$$

As the structure of asset markets affects the behavior of consumption and labor, the (in)-completeness of asset markets affects (in general) the behavior of capital in the two countries (because the marginal product of capital depends on labor).

It can however be shown that with identical preferences and technologies the (in)completeness of asset markets does not affect the behavior of worldwide aggregates of consumption, labor supply and capital in the approximate equilibrium. With fixed labor supplies, the condition that expected marginal products of capital are equated across countries implies that the capital stock in a given country can be expressed as a function of the world capital stock (and of expected future multiplicative technology shocks). Hence we get the result that with identical preferences and technologies and with fixed labor supplies, the approximate equilibrium has the property that the behavior of capital (and hence of output) in each of the two countries is the same with complete and with incomplete asset markets.

4. Stylized Facts

I evaluate the model with incomplete asset markets by comparing its predictions to the key stylized facts which characterize business cycles in industrialized countries (the following 'stylized facts' describe properties of detrended data).[17]

As documented in tables 15-17 for quarterly data, output, private consumption and investment are positively correlated across countries; consumption seems to be less correlated across countries than output. The averages of the cross-country correlations of quarterly growth rates of total private consumption, GNP and fixed investment in a sample countries consisting of the US, Japan, France, Britain, Italy and Canada during the period 1971:2-88:1 are 0.22, 0.26 and 0.24 respectively. When log

[17]See e.g. Backus & Kehoe (1989 a,b); Backus, Kehoe & Kydland (1989); Stockman & Tesar (1990) for statistical descriptions of salient features of international business cycles.

consumptions, log GNP and log investment are detrended using a linear time trend, the corresponding average cross-country correlations are 0.29, 0.40 and 0.42 respectively.

Within a given country the correlation of consumption and hours worked is low. Table 18 reports correlations between detrended consumption and hours for the US, Japan, France, Britain, Italy and Canada. The average of the within-country correlations of consumption and hours is 0.21.

Additional stylized facts which are of interest in evaluating the model developed in this paper are:

Within a given country, investment and consumption are procyclical; net exports are countercyclical; consumption tends to be less volatile than output, while investment is more volatile than output. For the US economy Backus, Kehoe & Kydland (1989) report that the percentage standard deviations of Hodrick-Prescott (1980) detrended quarterly output, consumption (non-durables and services) and fixed investment are 1.74, 0.86 and 5.51 respectively (in %).

The discussion in the next section centers mostly on the behavior of consumption, as the effect of the incompleteness of asset markets is strongest for that variable, and on the correlations of investment of output across countries, because the international correlation of investment and output differs significantly according to whether multiplicative or additive shocks are the source of fluctuations.

5. Simulations of the Model

5.1. *The Parameters of the Model*

I use simulations in order to investigate the properties of the model. These simulations focus on the symmetric case where the two countries have the same utility and productions functions, where the steady state values of the shocks which affect these countries are the same and where the steady state values of the net asset positions of the two countries are zero.

5.1.1. **Preference and Technologies**

The production function f(K,L) is assumed to be Cobb-Douglas:

$$f(K^i,L^i)=(K^i)^{\eta}*(L^i)^{1-\eta} \text{ with } 0\leq\eta\leq1 \quad . \tag{20}$$

Concerning the utility function u, a standard Real Business Cycle specification is adopted:[18]

$$u(c,L)=(1/(1-\sigma))*(c*e^{-\psi(L)})^{1-\sigma}, \qquad \text{where } \sigma>0,\ \sigma\neq1 \tag{21}$$

and ψ is an increasing function. One easily verifies that the concavity of u in c and L requires that $\nu_2+(\sigma-1)*\nu_1>0$ and $\sigma*\nu_2+(\sigma-1)*\nu_1>0$, where $\nu_1\equiv\psi'(L)*L$ and $\nu_2\equiv\psi''*L/\psi'$. Note that the steady state condition (10 d) implies (for the production function specified in (21)) that $\nu_1=(1-\eta)/(c/y)$; where $c/y = 1 - (g/y) - d*K/y = 1 - \gamma - d*(\alpha/r+d)$.

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[18]See King, Plosser & Rebelo (1990); Rotemberg & Woodford (1989).

ν_2 can be interpreted as a labor supply parameter:[19] holding constant country i's marginal instantaneous utility of consumption in period t ($u_c(c_t, L_t, \tau_t)$) and holding fixed $E_t V_{t+1}$, the elasticity of country i's labor supply with respect to its real wage rate (its marginal product of labor) is $els \equiv \sigma/(\sigma^* \nu_2 + (\sigma-1)^* \nu_1)$.

Holding constant the life-time utilities and the labor supplies of a country in periods t and t+1, $(1/\sigma)$ is its intertemporal elasticity of substitution between consumption in t and t+1. σ also affects the way in which consumption and hours worked interact in the instantaneous utility function u: the elasticity of u_1 with respect to hours worked is $-(1-\sigma)^* \nu_1$. By assumption ν_1 is positive. Hence consumption and hours are substitutes in the instantaneous utility function $u(c,L,\tau)$ if $\sigma<1$ and they are complements when $\sigma>1$. In the simulations, I focus on cases where consumption and hours are complements.

The function $\beta(c,L,\tau)$ is assumed to be of the form

$$\beta(c,L,\tau)=\beta(u(c,L,\tau)) \text{ with } \partial^2\beta/\partial u^2=0. \tag{22}$$

The elements of the matrices G_0, G_1, G_2 and H_0, H_1 and H_2 which determine the equilibrium behavior of the endogenous variables (see (11), (18)) can be expressed as functions of: r (the steady state interest rate), d (the depreciation rate of physical capital), η (the elasticity of output with respect to capital), the ratio $\gamma \equiv g/y$ (this share too is assumed to be the

[19]See the discussion in Rotemberg & Woodford (1989).

same for both countries), σ, $\varepsilon_{\beta,u} \equiv (\partial\beta/\partial u)*(u/\beta)$ and $\nu_2 \equiv \psi''*L/\psi'$.

To simulate the model, one thus has to choose values for r, d, η, σ, $\varepsilon_{\beta,u}$ and ν_2; in addition a stochastic process of the exogenous shocks has to be specified.

η (the elasticity of output with respect to capital) is set equal to 0.35. In all simulations, $(\partial\beta/\partial u)*(u/\beta)=-0.10$ is assumed. The baseline choice for σ is: $\sigma=2$ (Backus et al. (1989) use $\sigma=1.35$ as their baseline value). The remaining preference and technology parameters are chosen with the aim of comparing the predictions of the model to quarterly data; this motivates the following baseline choices for the steady state interest rate and the depreciation rate of capital: r=0.01 and d=0.025.

5.1.2. **The Stochastic Processes Followed by the Exogenous Shocks**

In the simulations, I abstract from taste shocks and I study the effects of one type of shock at a time, i.e. the model is either subjected to multiplicative or to additive shocks. These shocks are assumed to follow bivariate AR(1) processes:

$$z_t^{\theta}=RHO_{\theta}*z_{t-1}^{\theta}+\varepsilon_t^{\theta}, \quad \text{and} \qquad (23)$$
$$z_t^{g}=RHO_{g}*z_{t-1}^{g}+\varepsilon_t^{g}$$

where $z_t^{\theta}=(\Delta\theta_t^1,\ \Delta\theta_t^2)'$ and $z_t^{g}=(\Delta g_t^1,\ \Delta g_t^2)'$ while RHO_{θ} and RHO_g are 2x2 matrices, while ε_t^{θ} and ε_t^{g} are vectors of white noises with covariance matrices $VAR_{\theta} \equiv E\varepsilon_t^{\theta}\varepsilon_t^{\theta\prime}$ and $VAR_g \equiv E\varepsilon_t^{g}\varepsilon_t^{g\prime}$ respectively.

The effect of the incompleteness of asset markets on the cross-country correlation of consumption is likely to be affected by how strongly these shocks are correlated between countries: in the extreme case where the shocks affecting different countries are perfectly correlated, there is no scope for risk-sharing between these countries and hence it does not matter whether asset markets are complete or incomplete.
Hence it is important to estimate empirically how strong the correlation of the 'shocks' affecting different countries is.

Backus et al. (1989) present a model with multiplicative productivity shocks. They assume that the shocks affecting the two countries in their model follow an AR(1) process. Backus et al. fit an AR(1) process to detrended quarterly log Solow residuals for the US and an aggregate of three large European countries (Germany, France and Britain). Their estimates of RHO_θ and V_θ (see (23)) are:

$$RHO_\theta=\begin{bmatrix} .93 & .04 \\ .09 & .92 \end{bmatrix} \quad \text{and} \quad V_\theta=\begin{bmatrix} .0068^2 & .00001 \\ .00001 & .0078^2 \end{bmatrix}.$$

The correlation between the technology shocks affecting the two countries which is implied by these RHO_θ and V_θ matrices is 0.85. A correlation of 0.85 is much larger than the cross-country correlations which are documented below.

Unfortunately, Backus et al. do not say what data they used for the estimation of the Solow residuals; their paper also fails to provide standard errors for the estimates of RHO_θ and V_θ. It would be useful to know these standard errors, because the cross-country correlations implied by the estimates of RHO_θ and V_θ provided by Backus et al. are extremely sensitive to the values of the elements of the RHO_θ matrix. It appears for example that when the off-diagonal elements of the RHO_θ matrix reported by Backus et al. are set equal to zero, the cross-country correlation of the technology shocks drops from 0.85 to 0.18, which is much more consistent with the cross-country correlations reported in the appendix.

Costello (1989) presents estimates of cross-country correlations of Solow residuals which differ strongly from those implied by the Backus et al. estimates of RHO_θ and V_θ. She studies productivity growth in five two-digit SIC industries in six of the G7 countries. Her paper suggests that the cross-country correlations of detrended Solow residuals in these industries are close to zero.

Table 19 presents estimates of the correlation of detrended aggregate quarterly Solow residuals in the the US, Japan, France, Italy and Canada. These residuals are calculated using the formula $\ln(\theta) = \ln(y) - \eta^*\ln(K) - (1-\eta)^*\ln(L)$, assuming that $\eta=0.35$ (y, K and L are GNP, the stock of physical capital and hours worked respectively).

The GNP data are taken from the OECD Quarterly National Accounts.

Two measures of labor inputs are used: (i) for the US, Japan and France data on total hours worked in the non-agricultural sector are used. For the US, data on total employee hours are taken from Citibase (series LPMHU). For Japan and France, an index of hours worked was constructed by multiplying the series 'employment in non-agricultural establishments' and 'weekly hours of work in the non-agricultural sector' from the Bulletin of Labour Statistics published by the International Labour Office (ILO).

(ii) The second labor measure is the employment data provided by the International Financial Statistics (IFS). This measure is used for the US, Japan, France, Italy and Canada. The IFS employment series measure total numbers of employees and workers (and not total *hours*).

Capital stocks were estimated using data on gross capital formation from the International Financial Statistics, assuming a depreciation rate of 10%

p.a. .[20]

Table 19 presents cross-country correlations for first differences of log Solow residuals and for linearly detrended log Solow residuals.

A two-sided test of the null-hypothesis that cross-country correlations of Solow residuals are zero rejects that hypothesis at the 10% level for half of the estimated correlation coefficients reported in table 19.

The arithmetic average of the correlation coefficient based on first differenced Solow residuals is 0.19; for first differences Solow residuals, the average cross-country correlation is 0.21.

Changing the technology parameter η does not have a strong effect on the estimated cross-country correlations: for η=0.25, the average cross-country correlations of first differences and of linearly detrended log Solow residuals for the countries in the sample are 0.17 and 0.13 respectively. For η=0.45, the corresponding average cross-country correlations are 0.21 and 0.29 respectively.

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[20]From the identity $K_{t+1}=(1-d)*K_t+I_t$, where 'd' is the depreciation rate, I is gross investment capital, capital stocks can be constructed for periods t=1,...,T if one has an estimate of the capital stock in the 'initial' period t=0. In a steady state where GNP grows at a constant rate 'g' and where the capital to GNP and the investment to GNP ratios are constant, we have that k=i/(g+d), where k and i are the steady state capital to GNP and the steady state investment to GNP ratios respectively. I estimate the 'initial' capital stock by $\hat{K}_0 \equiv (\hat{i}/(\hat{g}+d))*\overline{Y}_0$, where $\hat{g}$ is the average GNP growth rate during the sample period and $\overline{Y}_0$ is the average GNP during the first four quarters of the sample, while $\hat{i} \equiv (1/T)*\sum I_t/Y_t$ is the sample average of the investment to GNP ratio.

Table 20 estimates trivariate AR(1) processes using detrended log Solow residuals for the US, Japan and France (the Solow residuals are constructed using η=0.35).
The autocorrelation (RHO) matrices which are estimated for first differences of log Solow residuals, are never significantly different from zero.
Almost all off-diagonal elements of the estimated autocorrelation matrices which are estimated using linearly detrended log Solow residuals fail to be significantly different from zero at conventional significance levels.
In the simulations presented below, I therefore assume that the off-diagonal elements of the 'RHO' matrix are zero.

5.2. *Simulation Results: Fixed Labor Supplies*

In the first set of simulations, reported in tables 22-27 I assume that labor supplies are fixed. I start the simulations of the model with this case because of the strong empirical evidence according to which labor supply elasticities are small (particularly for males).[21]
Intuitively we expect that the effects of limitations on international asset markets are stronger, the smaller the cross-country correlations of the exogenous shocks. The simulation in tables 22-27 show that when labor supplies are fixed and when shocks are not correlated across countries, the incompleteness of asset markets has a strong effect on the correlations of consumption and of life-time utilities across countries: in all the cases shown in tables 22-27, the cross-country correlation of consumption in the

[21] See Pencavel (1986).

presence of incomplete asset markets is smaller than 0.25. Furthermore the correlation between the expected life-time utility levels of the two countries tends to be *negative*. In the presence of complete asset markets, consumption and expected lifetime utility levels would be perfectly correlated across countries (given the assumptions of identical preferences in the two countries and fixed labor supplies).

As noted above, when both countries have identical preferences and technologies and when labor supplies are fixed, the approximate equilibrium has the property that the behavior of physical capital is the same with complete and with incomplete capital markets. The following discussion of the differences between the effects of multiplicative and additive shocks on investment and output applies thus to both asset market structures. The simulations reveal striking differences between the effects of multiplicative and additive shocks. When the model is subjected to multiplicative technology shocks (tables 22-24), investment in a given country tends to be very volatile and in addition investment tends to be negatively correlated across countries, unless the serial correlation of the technology shocks is low; this can be explained by the fact that when a persistent (positively serially correlated) country-specific multiplicative technology shock occurs in one of the countries, the increase in the marginal product of capital which results from the shock induces a strong increase in that country's investment while the other country's investment falls. This increase in physical investment in the country affected by a serially correlated positive multiplicative shock increases the country's demand for imports and thus implies that the trade balance is countercyclical.

When the model is subjected to multiplicative shocks, output is negatively correlated across countries, unless the serial correlation of the technology shocks is low. Simulations (not reported in the appendix) of the fixed labor supply model show that in order to get positive cross-country correlations of investment in the presence of serially correlated technology shocks, one has to assume that the multiplicative technology shocks are strongly correlated across countries. The same thing is true for output but the cross-country correlations of the multiplicative shocks required to make output positively correlated across countries seem to be lower than those needed to get positive cross-country correlations of investment.

When the fixed labor supply model is subjected to additive shocks (tables 25-27), then investment (and hence capital) is perfectly correlated across countries: as the marginal product of capital in the two countries is unaffected by additive technology shocks, the equalization of the marginal products of capital across the two countries (see (19)) implies that - with fixed labor supplies - the capital stocks of the two countries are perfectly correlated. An additive shock affecting one of the countries induces changes in the capital stocks of all countries in the world which have the same sign (provided the shock affects the world interest rate). Even with additive shocks which are not correlated across countries, the cross-country correlation of output ($y_t^i=\theta_t^i*f(K_t^i,L_t^i)-g_t^i$) is positive (although it is relatively small). Note however that if one interprets the *additive* shocks g_t^1 and g_t^2 as government consumption financed by lump-sum taxes (according to this interpretation $g_t^i>0$), then the output of country 'i' is given by $q_t^i \equiv \theta_t^i*f^i(K_t^i)$.

This implies that if government consumption (financed by lump-sum taxes) is the only source of fluctuations and if labor supplies are fixed, then both investment *and* output are perfectly correlated across countries. A drawback of a model based on additive shocks is that these shocks induce procyclical behavior of the trade balance.

5.3. *Simulation Results: Variable Labor Supplies*

Simulation results for the model with variable labor supplies and multiplicative shocks are presented in table 28.

The table presents simulation results for a grid of values of the intertemporal elasticity of substitution parameter σ and the labor supply elasticity els.[22]

As labor is immobile internationally, non-separabilities between consumption and work effort reduce the international covariation of consumption.[23] When the labor supply elasticity is sufficiently low and/or σ is sufficiently close to unity,[24] the simulation results are similar to those which obtain with fixed labor supplies; in particular, cross-country correlations of consumption are much lower with incomplete asset markets, than with complete markets.

An increase in σ strengthens the non-separability between consumption and

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[22]The following values of these parameters are considered: σ=1.1, 2, 3, 5, 7 and els=0.1, 0.5, 1.0, 2.0.

[23]See Devereux, Gregory & Smith (1991) for detailed discussions of this point.

[24]For σ=1, the period utility function specified in (21) becomes $u=\ln(c)-\psi(L)$, i.e. it becomes separable in consumption and hours worked.

hours (provided that $\sigma>1$) and holding constant the labor supply elasticity it therefore reduces the cross-country correlation of consumption. Likewise, increases in the labor-supply elasticty reduce the cross-country correlation of consumption (holding constant σ).

By selecting values of σ and els which are sufficiently large, the cross-country correlation of consumption can be made rather small, and that even when asset markets are complete; it appears however that this can only be achieved by letting consumption and hours be strongly correlated within the same country, which is inconsistent with the data.

We see that for $\sigma=1.1$ and els=0.1, the cross-country correlations of consumption with complete and with incomplete asset markets are 0.99 and 0.30 respectively, and that the correlation of consumption and hours within the same country is 0.18 with complete and 0.11 with incomplete asset markets. Increasing σ and els to $\sigma=2$ and els=2 reduces the cross-country correlation of consumption to 0.07 with complete and to 0.02 with incomplete asset markets. We see however that the within-country correlation of consumption and hours now becomes 0.77 with complete and 0.67 with incomplete asset markets, which is too large compared to the average within-country correlation observed in the data.

The simulations results therefore confirm the finding of Gregory et al. (1991) that non-separabilities between consumption and labor allow to strongly reduce the cross-country correlation of consumption, but they suggest that non-separabilities do not allow to reduce the cross-country correlation of consumption to the level seen in the data, while ensuring that within-country correlations of consumption and hours are small. A model with incomplete asset markets however is able to match both the low cross-country correlations of consumption and the low within-country correlations of consumption and hours, and that for 'realistic' cross-country correlations of technology shocks.

6. Summary

This chapter extends the Real Business Cycle (RBC) theory of international economic fluctuations by presenting an RBC model with incomplete international asset markets. In this model only debt contracts can be used for international capital flows.

The paper shows that with incomplete asset markets, the cross-country correlation of consumption can be markedly smaller than in the presence of complete asset markets. The simulations suggest that allowing for non-separabilities between consumption and labor in a one good international Real Business Cycle model with complete asset markets does not allow to reduce the cross-country correlation of consumption to the levels seen in the data while at the same time ensuring that the within-country correlations of consumption and hours are small. A model with incomplete asset markets however is able to match both the low cross-country correlations of consumption and the low within-country correlations of consumption and hours which are observed in the data, and that for 'realistic' cross-country correlations of technology shocks.

The essay shows also that a model which allows for *additive* technology shocks (which can be interpreted as shocks to government consumption) is better able to explain the observed positive correlations of investment and output across countries than standard business cycle theories in which *multiplicative* shocks to total (and marginal) factor productivities are the source of fluctuations.