Global Liquidity Traps

Robert Kollmann (*)
Université Libre de Bruxelles & CEPR

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This paper studies fluctuations of interest rates, inflation and output in a two-country New Keynesian business cycle model with a zero lower bound (ZLB) constraint for nominal interest rates. The presence of the ZLB generates multiple equilibria driven by self-fulfilling changes in domestic and foreign inflation expectation. Each country randomly switches in and out of a liquidity trap. In a floating exchange rate regime, liquidity traps can either be synchronized or unsynchronized across countries. This is the case even if countries are perfectly financially integrated. By contrast, in a monetary union, self-fulfilling fluctuations in inflation expectations must be perfectly correlated across countries.

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(*) Address: Prof. R. Kollmann, Solvay Brussels School of Economics & Management, Université Libre de Bruxelles, CP114, 50 Av. Franklin D. Roosevelt, 1050 Brussels, Belgium
robert_kollmann@yahoo.com www.robertkollmann.com

1. Introduction

This paper studies fluctuations of inflation, real activity and the exchange rate in a two-country New Keynesian business cycle model with a zero lower bound (ZLB) for nominal interest rates. The recent experience of persistent low interest rates and low inflation in advanced economies has led to a resurgence of research on liquidity traps, defined at situations in which the interest rate is zero (or close to zero), so that monetary policy cannot stimulate real activity by lowering the policy rate (Keynes (1936), Hicks (1937)). See Krugman (1998), Benhabib et al. (2001a,b. 2002a,b) and Eggertsson and Woodford (2003) for seminal analyses of liquidity traps based on modern dynamic models.

One of the key insights of the recent literature is that the ZLB constraint gives rise to multiple equilibria, under the standard assumption that monetary policy follows an 'active' Taylor rule, i.e. a policy rule with a strong interest rate response when the inflation rate deviates from the central bank's inflation target. Benhabib et al. (2001a,b) show that the combination of the ZLB and a Taylor rule generates two steady states: in one steady state, the private sector expects the inflation rate to equal the target inflation rate, and the nominal interest rate is strictly positive; in the other steady state, the expected and actual inflation rates are smaller than the inflation target, and the nominal interest rate is zero.[1] Arifovic et al. (2018) and Aruoba et al. (2018) show, using closed economy models, that the combination of the ZLB and a Taylor rule also produces multiples *stochastic* equilibria in which the economy repeatedly and randomly enters and exit a liquidity trap. These stochastic fluctuations are driven by self-fulfilling changes in private sector inflation expectations, and they are consistent with rational expectations. Importantly, equilibria with recurrent ZLB episodes are possible without adverse exogenous shocks to productivity, government purchases or other 'fundamental' drivers of business cycles.

This paper contributes to the literature on 'beliefs-driven' liquidity traps by considering open economies. I demonstrate the existence of multiple equilibria in a New Keynesian two-country model with a ZLB and Taylor rules. The exchange rate regime is a key determinant of the international synchronization of liquidity traps. Under a floating exchange rate, a country can fall into a liquidity trap, regardless of whether the rest of the world is in a liquidity trap or not. This is the case even if countries are perfectly financially integrated. By contrast, in a monetary union, all member countries face a liquidity trap at the same time, and (in the absence of exogenous country-specific aggregate supply or demand shocks) self-fulfilling fluctuations in inflation expectations must be perfectly correlated across countries.

Several other recent studies have considered liquidity traps in open economies. However, those studies assume that a liquidity trap occurs when the economy is hit by a large exogenous temporary negative aggregate demand shock (such as an exogenous rise in household saving); the liquidity trap is assumed to end when the adverse aggregate demand shock subsides. See e.g. Jeanne (2009), Cook and Devereux (2013) and Acharya and Bengui (2018). That literature does not address the critical indeterminacy induced by the ZLB. By contrast to the paper here, the prior open economy literature does not consider stochastic equilibria with recurrent beliefs-driven recurrent liquidity traps. In multi-country models, adverse demand shocks hitting a large country tend to induce a liquidity trap both in that country and in the rest of the world (e.g., Jeanne (2009)). By contrast, the analysis here suggests that when a liquidity trap is triggered by a

[1] Benhabib et al. (2001a) constructed *deterministic* equilibrium trajectories that originate close to the first steady state (away from the ZLB) and that converge to the liquidity trap. See Holden (2016) for a detailed analysis of multiple deterministic (perfect foresight) equilibrium trajectories induced by the presence of the ZLB.

self-fulfilling decline in inflation expectations, then cross-country spillovers are weak, and thus a liquidity trap in one part of the world does not induce a liquidity trap in the rest of the world.

2. The model: two-country world with floating exchange rate

I consider a New Keynesian open economy model with a standard structure of goods, labor and financial markets (e.g., Kollmann (2001, 2002, 2004)). There are two countries, referred to as Home (H) and Foreign (F). This Section assumes a floating exchange rate (the case of a monetary union is discussed in the next Section). In each country there are: (i) a representative infinitely-lived household; (ii) a central bank; (iii) monopolistic firms that produce a continuum of differentiated tradable intermediate goods using domestic labor. (iv) competitive firms that bundle domestic and imported intermediates into a composite non-tradable final consumption good. Each country's household owns the domestic firms, and it supplies labor to those firms (labor is immobile internationally). The labor market is competitive. For analytical tractability, the model abstracts from physical capital. The Foreign country is a mirror image of the Home country. The following description focuses on the Home country. Analogous conditions describe the Foreign country. To focus on the role of self-fulfilling changes in expectations as drivers of economic fluctuations, I assume that there no exogenous shocks to technology, preferences or monetary policy. The specification of preferences of technologies described below (log utility, unit substitution elasticity between domestic and foreign intermediates, linear intermediate good production technologies) is chosen to simplify the analysis. The main results go through for more general specifications.

2.1. Home firms

The Home country final good is produced using the aggregate Cobb-Douglas technology $Z_{H,t}\equiv(Y_{H,t}^{H}/\xi)^{\xi}(Y_{H,t}^{F}/(1-\xi))^{1-\xi}$ where $Z_{H,t}$ is date t final good output in country H, while and $Y_{H,t}^{H}$ and $Y_{H,t}^{F}$ are composite domestic and imported intermediates used by country H, respectively. (The superscript on intermediate good quantities and prices denotes the country of origin, while the subscript indicates the destination country.) There is a bias towards using local inputs: $0.5<\xi<1$. Each country produces a distinct set of intermediates indexed by $s\in[0,1]$. (Intermediate good 's' produced by country H is distinct from intermediate 's' produced by country F.) The composite intermediate $Y_{H,t}^{k}$ is given by $Y_{H,t}^{k}\equiv\{\int_0^1(y_{H,t}^{k}(s))^{(\nu-1)/\nu}ds\}^{\nu/(\nu-1)}$ with $\nu>1$, for k=H,F, where $y_{H,t}^{k}(s)$ is the quantity of the variety input produced by country k that is sold to country H. Let $p_{H,t}^{k}(s)$ be price of that intermediate, in country H currency. Cost minimization in final good production implies: $y_{H,t}^{k}(s)=(p_{H,t}^{k}(s)/P_{H,t}^{k})^{-\nu}Y_{H,t}^{k}$ for k=H,F and $Y_{H,t}^{H}=\xi\cdot CPI_{H,t}\cdot Z_{H,t}/P_{H,t}^{H}$, $Y_{H,t}^{F}=(1-\xi)\cdot CPI_{H,t}\cdot Z_{H,t}/P_{H,t}^{F}$ where $P_{H,t}^{k}\equiv\{\int_{s=0}^{1}p_{H,t}^{k}(s)^{1-\nu}ds\}^{1/(1-\nu)}$ and $CPI_{H,t}\equiv(P_{H,t}^{H})^{\xi}(P_{F,t}^{H})^{1-\xi}$. $P_{H,t}^{k}$ is a price index of country k=H,F intermediates sold in country H. Perfect competition implies that the country H final good price is $CPI_{H,t}$ (its marginal cost). The technology of the firm that produces intermediate good s, in country H is: $y_{H,t}(s)=L_{H,t}(s)$, where $y_{H,t}(s)$ and $L_{H,t}(s)$ are the firm's output and labor input at date t. The firm's good is sold domestically and exported: $y_{H,t}(s)=y_{H,t}^{H}(s)+y_{F,t}^{H}(s)$.

The law of one price is assumed to hold: $p^H_{H,t}(s)=p^H_{F,t}(s)/S_t$, where S_t is the nominal exchange rate, defined as the Foreign currency price of Home currency (i.e. a rise in S_t is an appreciation of the Home currency). Thus, $P_{H,t}\equiv P^H_{H,t}=P^H_{F,t}/S_t$ and $P_{F,t}\equiv P^F_{F,t}=P^F_{H,t}\cdot S_t$.

Intermediate good producers face quadratic costs to adjusting their prices, in producer currency. The real profit, in units of the domestic final good, of the firm that produces Home intermediate good *s* is:

$$\pi_{H,t}(s)\equiv(p^H_{H,t}(s)-W_{H,t})[y^H_{H,t}(s)+y^H_{F,t}(s)]/CPI_{H,t}-\tfrac{1}{2}\psi\cdot([p^H_{H,t}(s)-\Pi\cdot p^H_{H,t-1}(s)]/P_{H,t-1})^2,\quad \psi>0$$

where $W_{H,t}$ is the nominal wage rate in country H. The last term in the profit equation is the price adjustment cost (expressed in final good units), where $\Pi>1$ is the central bank's gross inflation target. [2] The firm sets $p^H_{H,t}(s)$ to maximizes the present value of profits $E_t\sum_{\tau=0}^{\infty}\rho^H_{t,t+\tau}\pi_{H,t+\tau}(s)$, where $\rho^H_{t,t+\tau}$ is the Home household's intertemporal marginal rate of substitution in consumption between periods t and $t+\tau$. All Home intermediate good firms face identical decision problems, and they set identical prices: $p^H_{H,t}(s)=P_{H,t}$ $\forall$ s$\in$[0,1]. The labor input and output too are equated across these firms.

The Country H terms of trade and real exchange rate are $q_{H,t}\equiv S_tP_{H,t}/P_{F,t}$ and $RER_{H,t}\equiv S_t CPI_{H,t}/CPI_{F,t}$, respectively. Note that $RER_{H,t}=(q_{H,t})^{2\xi-1}$. Due to home bias ($2\xi$-1>0), the real exchange rate is an increasing function of the terms of trade. The real price of the domestic intermediate good, in units of final consumption, too is an increasing function of the terms of trade:

$$P_{H,t}/CPI_{H,t}=(q_{H,t})^{1-\xi}. \tag{1}$$

2.2. Household preferences and labor supply

The preferences of the representative Home household are described by $E_0\sum_{t=0}^{\infty}\beta^t U(C_{H,t},L_{H,t})$ where $C_{H,t}$ and $L_{H,t}$ are final consumption and aggregate hours worked, respectively. $0<\beta<1$ is the household's subjective discount factor and $U(C_{H,t},L_{H,t})=\ln(C_{H,t})-\frac{1}{1+1/\eta}(L_{H,t})^{1+1/\eta}$, with $\eta>0$, is the agent's period utility function. The household equates the marginal rate of substitution between leisure and consumption to the real wage rate, which implies

$$(1/C_{H,t})(W_{H,t}/CPI_{H,t})=(L_{H,t})^{1/\eta}. \tag{2}$$

2.3. Financial markets

The model assumes complete international financial markets, so that consumption risk is efficiently shared across countries. In equilibrium, the ratio of Home to Foreign households' marginal utilities of consumption is, thus, proportional to the Home real exchange rate (Kollmann, 1991, 1995; Backus and Smith, 1993). With log utility, this implies that relative Home/Foreign consumption is inversely proportional to the Home real exchange rate:

[2] The price adjustment cost is assumed to be a quadratic function of $p^H_{H,t}(s)-\Pi\cdot p^H_{H,t-1}(s)$ (and not of $p^H_{H,t}(s)-p^H_{H,t-1}(s)$, as e.g. in the Rotemberg (1982) cost adjustment original specification). This ensures that the long-run Phillips curve is vertical.

$C_{H,t}/C_{F,t}=\Lambda/RER_{H,t}$, where Λ is a date- and state-invariant term that reflects the (relative) initial wealth of the two countries. I assume that the two countries have the same initial wealth, i.e. Λ=1. Thus:

$$C_{H,t}/C_{F,t}=1/RER_{H,t}. \tag{3}$$

There are also markets for one-period nominal bond (in zero net supply) denominated in Home and Foreign currency, respectively. The nominal interest rate on the Home currency bond is $i_{H,t+1}$ (between periods *t* and *t*+1). The Home household's Euler equation for this bond is:

$$(1+i_{H,t+1})E_t\beta(C_t/C_{t+1})/\Pi^{CPI}_{H,t+1}=1, \tag{4}$$

where $\Pi^{CPI}_{H,t+1}\equiv CPI_{H,t+1}/CPI_{H,t}$ is the Home gross CPI inflation rate (between *t* and *t*+1).

2.4. Monetary policy: floating exchange rate

The central bank of country *k*=H,F sets the interest rate in local currency, $i_{k,t+1}$, according to an interest rate feedback rule that targets the domestic PPI inflation rate $\Pi_{k,t}\equiv P_{k,t}/P_{k,t-1}$, subject to the ZLB constraint $i_{k,t+1}\geq 0$. Specifically:

$$1+i_{k,t+1}=Max\{1,\Pi/\beta+(\gamma_\pi/\beta)\cdot(\Pi_{k,t}-\Pi)\},\ \gamma_\pi>1 \qquad \text{for } k=\text{H,F} \tag{5}$$

where Π/β is the value of the gross nominal interest rate that obtains when the inflation rate equals the central bank's inflation target. The 'Taylor principle' holds: $\gamma_\pi>1$, i.e. a rise in domestic inflation by 1 percentage point (ppt) triggers a rise of the policy rate by more than 1 ppt.

2.5. Market clearing

Market clearing in the Home labor and final good markets requires $L_{H,t}=\int_{s=0}^{1}L_{H,t}(s)\,ds$ and $Z_{H,t}=C_{H,t}$, respectively. Home real GDP equals aggregate intermediate good output, $Y_{H,t}=L_{H,t}$. Markets for individual intermediates clear as intermediate good firms meet all demand at posted prices. This implies $Y_{H,t}=Y^H_{H,t}+Y^H_{F,t}$, i.e. aggregate intermediate good output equals the sum of aggregate domestic and foreign intermediate good demand. Using the domestic and foreign intermediate good demand functions described above, this condition can be expressed as $Y_{H,t}=\xi CPI_{H,t}C_{H,t}/P_{H,t}+(1-\xi)CPI_{F,t}C_{F,t}/(P_{H,t}S_t)$. The risk sharing condition (3) gives $CPI_{H,t}C_{H,t}=CPI_{F,t}C_{F,t}/S_t$, i.e. consumption spending is equated across countries. Thus, Home intermediate good market clearing implies $Y_{H,t}=CPI_{H,t}C_{H,t}/P_{H,t}$, which shows that Home nominal GDP equals nominal consumption spending: $P_{H,t}Y_{H,t}=CPI_{H,t}C_{H,t}$. Thus, the trade balance is zero. Using (1), the market clearing condition for Home intermediates can also be expressed as

$$Y_{H,t}=C_{H,t}\cdot(q_{H,t})^{\xi-1}. \tag{6}$$

2.6. Solving the model

The Foreign intermediate good market clearing condition is $Y_{F,t}=C_{F,t}\cdot(q_{H,t})^{1-\xi}$. Thus, $Y_{H,t}/Y_{F,t}=(C_{H,t}/C_{F,t})\cdot(q_{H,t})^{2\xi-2}$. The risk-sharing condition (3) implies $C_{H,t}/C_{F,t}=(q_{H,t})^{1-2\xi}$. Hence,

$$q_{H,t}=Y_{F,t}/Y_{H,t}\,, \tag{7}$$

i.e. the Home terms of trade equal the inverse of Home relative real GDP; therefore, nominal Home GDP equals nominal Foreign GDP, expressed in the same currency. Using (6) and (7) one can express Home aggregate consumption as a function of Home and Foreign GDP:

$$C_{H,t}=(Y_{H,t})^{\xi}(Y_{F,t})^{1-\xi}\text{; similarly, } C_{F,t}=(Y_{H,t})^{1-\xi}(Y_{F,t})^{\xi}. \tag{8}$$

Because $P_{H,t}Y_{H,t}=CPI_{H,t}C_{H,t}$, the growth of nominal consumption spending equals the growth of nominal GDP. Thus, the Home Euler equation (4) can be written as

$$(1+i_{H,t+1})E_t\beta/(\Pi_{H,t+1}Y_{H,t+1}/Y_{H,t})=1. \tag{9}$$

The analogous Euler equation in country Foreign is:

$$(1+i_{F,t+1})E_t\beta/(\Pi_{F,t+1}Y_{F,t+1}/Y_{F,t})=1. \tag{10}$$

Following much of the previous literature on macro models with a ZLB constraint (see Holden (2016) for detailed references), I linearize all equations, with the exception of the interest rate rule (5). This allows to capture the macroeconomic effects of the occasionally binding ZLB constraint, while keeping analytical tractability.

I take a linear approximation around a steady state in which (in both countries) the gross inflation rate equals the inflation target Π; the corresponding gross interest rate is $1+i_k=\Pi/\beta$ for k=H,F. Let $\hat{x}_t\equiv(x_t-x)/x$ denote the relative deviation of a variable x_t from its steady state value x (variables without time subscript denote steady state values). Linearization of (7)-(10) gives

$$\widehat{q_{H,t}}=\widehat{Y_{F,t}}-\widehat{Y_{H,t}}; \tag{11}$$

$$\widehat{C_{H,t}}=\xi\cdot\widehat{Y_{H,t}}+(1-\xi)\cdot\widehat{Y_{F,t}} \quad\text{and}\quad \widehat{C_{F,t}}=(1-\xi)\cdot\widehat{Y_{H,t}}+\xi\cdot\widehat{Y_{F,t}}; \tag{12}$$

$$\widehat{1+i_{k,t+1}}=E_t\{\widehat{\Pi_{k,t+1}}+\widehat{Y_{k,t+1}}-\widehat{Y_{k,t}}\},\ k\text{=H,F}. \tag{13}$$

Linearizing the first-order condition of the intermediate good firms' decision problem in country k=H,F gives a 'forward-looking' Phillips equation: $\widehat{\Pi_{k,t}}=\kappa_w\cdot\widehat{W_{k,t}/P_{k,t}}+\beta E_t\widehat{\Pi_{k,t+1}}$, where $\kappa_w>0$ is a coefficient that is a decreasing function of the price adjustment-cost parameter ψ. Thus, the PPI inflation rate is an increasing function of the real product wage (and thus of real marginal cost), and of expected future PPI inflation. It is easy to see that $W_{k,t}/P_{k,t}=(Y_{k,t})^{1+1/\eta}$.[3] Thus, the country k Phillips equation can be expressed as

$$\widehat{\Pi_{k,t}}=\kappa\cdot\widehat{Y_{k,t}}+\beta E_t\widehat{\Pi_{k,t+1}}\,,\quad k\text{=H,F} \tag{14}$$

with $\kappa\equiv\kappa_w\cdot(1+1/\eta)>0$. Hence, PPI inflation in country k is an increasing function of local GDP and of the expected future local PPI inflation rate.

For convenience, I also express the interest rate rule (5) using 'hatted' variables. This gives:

[3] Note that $W_{k,t}/P_{k,t}=(W_{k,t}/CPI_{k,t})/(P_{k,t}/CPI_{k,t})$. The labor supply equation (2) implies $W_{k,t}/CPI_{k,t}=C_{k,t}(Y_{k,t})^{1/\eta}$ and so $W_{k,t}/P_{k,t}=C_{k,t}(Y_{k,t})^{1/\eta}(q_{k,t})^{\xi-1}$ which gives $W_{k,t}/P_{k,t}=(Y_{k,t})^{1+1/\eta}$ because of (6).

$$\widehat{(1+i_{k,t+1})} = Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi_{k,t}}\}. \tag{15}$$

Note that $\widehat{(1+i_{k,t+1})}$ is a *non-linear* function of domestic PPI inflation. Country *k* hits the ZLB bound when $\gamma_\pi \widehat{\Pi_{k,t}} \leq \{(\Pi-\beta)/\Pi\} < 0$

Combining the Euler equation (13) and the interest rate rule (15), and substituting out GDP using the Phillips equation (14) gives the following non-linear equation that governs the dynamics of PPI inflation in country *k*:

$$Max\{-(\Pi-\beta)/\Pi, \gamma_\pi \cdot \widehat{\Pi_{k,t}}\} + \tfrac{1}{\kappa}\widehat{\Pi_{k,t}} = (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi_{k,t+1}} - \tfrac{\beta}{\kappa}E_t\widehat{\Pi_{k,t+2}}\text{ , } k\text{=H,F.} \tag{16}$$

This equation links date *t* PPI inflation in country *k* to expected future inflation in the *same* country. Given a PPI process that solves (16), we can determine country *k* GDP, consumption and terms of trade using (14), (12) and (11).

2.7. Multiple equilibria

The inflation equation (16) has multiple bounded solutions. As shown below, there exist solutions with recurrent ZLB episodes. Note that inflation equation (16) for country Home is autonomous from the corresponding inflation equation for Foreign. This implies that, in the floating exchange rate regime assumed here, the relation between Home and Foreign inflation is indeterminate: any correlation between Home and Foreign inflation is possible, and thus the cross-country correlation of ZLB too is indeterminate. This is the case even if countries are perfectly financially integrated (as assumed in the present model).

Given our assumption that the Taylor principle holds (γ_π>1), (16) is solved by two constant (steady state) inflation rates: $\widehat{\Pi_k}=0$ and $\widehat{\Pi_k}=-(\Pi-\beta)/\Pi$. The ZLB binds in the second steady state. [4] The ZLB in Home can bind forever, irrespective of whether the Foreign ZLB constraint binds.

There also exist equilibria with time-varying stochastic inflation. First of all, there exist multiple stochastic solutions for country *k* inflation that fluctuate in the vicinity of $-(\Pi-\beta)/\Pi$, such that the ZLB binds always. When the ZLB binds $\forall t$ then $\tfrac{1}{\kappa}\widehat{\Pi_{k,t}} = (1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi_{k,t+1}} - \tfrac{\beta}{\kappa}E_t\widehat{\Pi_{k,t+2}}$ must hold (from (16)). The state-space form of this expectational difference equation has one eigenvalue strictly greater than unity and one eigenvalue strictly smaller than unity (under our assumptions that κ>0, 0<β<1), which entails the existence of multiple bounded solutions (see Blanchard and Kahn (1980), Prop. 3).

By contrast, an equilibrium in which the ZLB never binds must exhibit a constant inflation rate: $\widehat{\Pi_{k,t}}=0\ \forall t$. Note that when the ZLB never binds then equation (16) gives $(\gamma_\pi+\tfrac{1}{\kappa})\widehat{\Pi_{k,t}}=(1+\tfrac{1+\beta}{\kappa})E_t\widehat{\Pi_{k,t+1}}-\tfrac{\beta}{\kappa}E_t\widehat{\Pi_{k,t+2}}$. The Taylor principle $\gamma_\pi>1$ ensures that both eigenvalues of the corresponding state-space form are outside the unit circle, and so this equation has a *unique* non-explosive solution given by $\widehat{\Pi_{k,t}}=0\ \forall t$.

Stochastic inflation fluctuations with occasional binding ZLB constraint

[4] This finding is in line with the result by Benhabib et al. (2001a,b) that the combination of the ZLB and the Taylor rule produces two steady states, and that the ZLB binds in one of these steady states.

In addition, there exist equilibria that feature stochastic inflation fluctuations, with an *occasionally* binding ZLB. Those equilibria seem especially relevant empirically, and I thus focus the following discussion on these equilibria.

Following Arifovic et al. (2018) and Aruoba et al. (2018) (who studied multiple equilibria in closed economies with a ZLB), I assume that Home and Foreign inflation follows a Markov chain in which inflation takes two possible values: $\widehat{\Pi_{k,t}} \in \{\widehat{\Pi_U}, \widehat{\Pi_D}\}$ such that

$$\gamma_\pi \widehat{\Pi_D} < -(\Pi - \beta)/\Pi < \gamma_\pi \widehat{\Pi_U}. \tag{17}$$

Thus, the ZLB binds when low inflation $\widehat{\Pi_D}$ is realized; it does not bind when higher inflation rate $\widehat{\Pi_U}$ obtains. For simplicity, I focus here on equilibria in which the transition probabilities of the Markov chain are symmetric across countries, and in which the conditional probabilities of country k inflation at t+1 only depend on date t inflation in that same country:

$$p_{i,j} \equiv \text{Prob}(\widehat{\Pi_{k,t+1}} = \widehat{\Pi_j} | \widehat{\Pi_{k,t}} = \widehat{\Pi_i}) = \text{Prob}(\widehat{\Pi_{k,t+1}} = \widehat{\Pi_j} | I_t) \text{ for } i,j \in \{\text{U;D}\}, \tag{18}$$

with $0 \le p_{i,j} \le 1$ and $p_{i,U} + p_{i,D} = 1$, where I_t is the date t information set (consisting of all contemporaneous and lagged variables). [5]

Then the conditional expected values of future inflation in country k is solely a function of current domestic inflation: $E_t \widehat{\Pi_{k,t+s}} = E[\widehat{\Pi_{k,t+s}} | \widehat{\Pi_{k,t}}]$ for s>0. (18) implies that

$$E(\widehat{\Pi_{k,t+1}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_U}) = p_{U,U} \widehat{\Pi_U} + p_{U,D} \widehat{\Pi_D} \text{ and } E(\widehat{\Pi_{k,t+1}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_D}) = p_{D,U} \widehat{\Pi_U} + p_{D,D} \widehat{\Pi_D};$$

$$E(\widehat{\Pi_{k,t+2}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_U}) = (p_{U,U} p_{U,U} + p_{U,D} p_{D,U}) \widehat{\Pi_U} + (p_{U,U} p_{U,D} + p_{U,D} p_{D,D}) \widehat{\Pi_D};$$

$$E(\widehat{\Pi_{k,t+2}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_D}) = (p_{D,U} p_{U,U} + p_{D,D} p_{D,U}) \widehat{\Pi_U} + (p_{D,U} p_{U,D} + p_{D,D} p_{D,D}) \widehat{\Pi_D}.$$

A Markov-chain equilibrium with an occasionally binding ZLB is defined by inflation rates $\widehat{\Pi_U}, \widehat{\Pi_D}$ and transition probabilities $p_{U,U}, p_{D,D}$ such that inequalities in (17) are satisfied and the following conditions hold (that follow from price equation (16)):

$$(\gamma_\pi + \tfrac{1}{\kappa}) \widehat{\Pi_U} = (1 + \tfrac{1+\beta}{\kappa}) E[\widehat{\Pi_{k,t+1}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_U}] - \tfrac{\beta}{\kappa} E[\widehat{\Pi_{k,t+2}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_U}], \; k\text{=H,F}; \tag{19}$$

$$-(\Pi - \beta)/\Pi + \tfrac{1}{\kappa} \widehat{\Pi_D} = (1 + \tfrac{1+\beta}{\kappa}) E[\widehat{\Pi_{k,t+1}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_D}] - \tfrac{\beta}{\kappa} E[\widehat{\Pi_{k,t+2}} | \widehat{\Pi_{k,t}} = \widehat{\Pi_D}], k\text{=H,F}. \tag{20}$$

Equations (19) describes price setting when the ZLB constraint is slack, while (20) pertains to price setting in the liquidity trap.

Model calibration

One period in the model is taken to represent one quarter in calendar time. I set β=0.9975, which implies a 1% per annum steady state riskless real interest rate, consistent with the low real interest rates observed in advanced economies during the last two decades. The quarterly gross inflation target is set at Π=1.005, in line with the (roughly) 2% p.a. inflation target of the ECB and other central banks in advanced countries. The inflation coefficient of the interest rate rule is

[5] Specification (18) allows for correlated Home and Foreign inflation rates. Let $p_{im,jn} \equiv \text{Prob}(\widehat{\Pi_{H,t+1}} = \widehat{\Pi_j}, \widehat{\Pi_{F,t+1}} = \widehat{\Pi_n} | \widehat{\Pi_{H,t}} = \widehat{\Pi_i}, \widehat{\Pi_{F,t}} = \widehat{\Pi_m})$ for i,j,m,n $\in$\{U;D\} be the *joint* Home and Foreign inflation transition probabilities. Home and Foreign inflation rates are uncorrelated when $p_{im,jn} = p_{i,j} p_{m,n}$. The cross-country inflation correlation differs from zero when $p_{im,jn} \ne p_{i,j} p_{m,n}$.

set at the conventional value $\Gamma_\pi = 1.5$. The slope of the Phillips curve is set at the standard value $\kappa = 0.1$.

The existence of an equilibrium with an occasionally binding ZLB requires highly persistent inflation processes, i.e. values of p_{UU} and p_{DD} close to unity. Assume, for example, $p_{UU} = p_{DD} = 0.95$. Then (17),(19) and (20) are solved by $\widehat{\Pi_D} = -0.008148$, $\widehat{\Pi_U} = -0.001370$. This corresponds to annualized inflation rates in the 'low' and 'high' inflation states of -1.27% and 1.44%, respectively.

3. Monetary Union

This Section studies a variant of the two-country model with a monetary union. The two countries now have the same currency, and there is a common central bank that sets the union-wide interest rate, targeting the union-wide inflation rate $\Pi_t \equiv (P_{H,t} + P_{F,t})/(P_{H,t-1} + P_{F,t-1})$, using the policy rule

$$1 + i_{k,t+1} = Max\{1, \Pi/\beta + (\gamma_\pi/\beta)\cdot(\Pi_t - \Pi)\},\ \gamma_\pi > 1 \tag{21}$$

Union-wide aggregate real GDP is $Y_t \equiv Y_{H,t} + Y_{F,t}$. Up to a linear approximation (around a symmetric steady state) $\widehat{\Pi}_t = \frac{1}{2}\widehat{\Pi_{H,t}} + \frac{1}{2}\widehat{\Pi_{F,t}}$ and $\widehat{Y}_t = \frac{1}{2}\widehat{Y_{H,t}} + \frac{1}{2}\widehat{Y_{F,t}}$. The first-order conditions of households and firms derived above continue to holds in a monetary union. Adding together the Home and Foreign Phillips curves (14) gives a union-wide Phillips curve that links union-wide GDP and inflation:

$$\widehat{\Pi}_t = \kappa \cdot \widehat{Y}_t + \beta E_t \widehat{\Pi_{t+1}},\quad k = \text{H,F}. \tag{22}$$

The presence of the ZLB constraint now gives rise to multiple equilibria characterized by bounded stochastic fluctuations of *union-wide* inflation and output, with an *occasionally* binding ZLB. These fluctuations are driven by self-fulfilling changes in expected union-wide inflation. However, there is no scope for *country-specific* inflation bubbles, in a monetary union: inflation must be perfectly correlated across countries.

To understand this result, note that when the two countries have the same currency, the nominal exchange rate is unity, $S_t = 1$, and so the Home terms of trade equal the ratio of Home to Foreign producer prices: $q_{H,t} \equiv P_{H,t}/P_{F,t}$. Thus, the growth rate of the terms of trade is the difference between Home and Foreign PPI inflation rates:

$$\widehat{q_{H,t}} - \widehat{q_{H,t-1}} = \widehat{\Pi_{H,t}} - \widehat{\Pi_{F,t}}. \tag{23}$$

Subtracting the Foreign Phillips equation from the Home Phillips equation (see (14)) gives $\widehat{\Pi_{H,t}} - \widehat{\Pi_{F,t}} = \kappa \cdot (\widehat{Y_t^H} - \widehat{Y_t^F}) + \beta E_t\{\widehat{\Pi_{H,t+1}} - \widehat{\Pi_{F,t+1}}\}$. Using (11) and (23), this gives the following law of motion of the terms of trade: $\widehat{q_{H,t}} - \widehat{q_{H,t-1}} = -\kappa \cdot \widehat{q_{H,t}} + \beta E_t\{\widehat{q_{H,t+1}} - \widehat{q_{H,t}}\}$. The state-space form of this expectational difference equation is:

$$\begin{bmatrix} E_t \widehat{q_{H,t+1}} \\ \widehat{q_{H,t}} \end{bmatrix} = \begin{bmatrix} (1+\beta+\kappa)/\beta & -1/\beta \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \widehat{q_{H,t}} \\ \widehat{q_{H,t-1}} \end{bmatrix}. \tag{24}$$

Given our assumptions that $0 < \beta < 1$ and $\kappa > 0$, this system has one eigenvalue stricter greater than one, and one eigenvalue smaller than one. There is one non-predetermined variable at date *t*,

$\widehat{q_{H,t}}$, and one predetermined variable, $\widehat{q_{H,t-1}}$. Thus, there is a unique bounded solution for the terms of trade, given by

$$\widehat{q_{H,t+1}}=J\cdot\widehat{q_{H,t}}, \tag{25}$$

where $0<J<1$ is the smaller of the two eigenvalues of the 2x2 matrix in (24).

Thus, the terms of trade in the initial period, say t=0, $q_{H,0}$ pins down the path of the terms of trade in all subsequent periods. Beliefs-driven changes in inflation in later periods have no effect on the trajectory of the terms of trade. Thus, beliefs-driven inflation changes must be perfectly correlated across countries.

In the long-run, the terms of trade asymptotes to the steady state $\widehat{q_H}=0$.[6] Assume that, by period t, the terms of trade in have converged (sufficiently close) to steady state, so that $\widehat{q_{H,t}}=0$. Then, output and prices in the monetary union are equated across countries: $P_{H,t}=P_{F,t}$, $Y_{H,t}=Y_{F,t}$. (The model assumes symmetric countries, and it abstracts from exogenous shocks to productivity or preferences. Country-specific productivity or demand shocks can obviously induce deviations between Home and Foreign prices.)

4. Conclusion

This paper studies fluctuations of inflation and output in a two-country New Keynesian business cycle model with a zero lower bound (ZLB) constraint for nominal interest rates. The presence of the ZLB generates multiple equilibria driven by self-fulfilling changes in domestic and foreign inflation expectation. Each country randomly switches in and out of a liquidity trap. In a floating exchange rate regime, the occurrence of liquidity traps can either be synchronized or unsynchronized across countries. This is the case even if countries are perfectly financially integrated. By contrast, in a monetary union, self-fulfilling fluctuations in inflation expectations must be perfectly correlated across countries.

[6] For the parameter values assumed in the simulations above, β=0.9975, κ=0.1, we have J=.73061. The implied half-life of the terms of trade is 2.2 quarters, and thus the terms of trade converge very fast to steady state.

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